

Pensieve header: Solving for V alone (with Kuno). Continues pensieve://Projects/WKO4.

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\People\\Kuno"];
<< FreeLie.m;
```

FreeLie` implements / extends

{\*, +, \*\*, \$SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords, AllWords, Arbitrator, AS, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ℓ, Δ, σ, ħ, ↦, ↪}.

FreeLie` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150814.

```
In[*]:= x = LW@"x"; y = LW@"y";
bch = BCH[x, y]
```

Out[\*]=

$$LS\left[\overline{x} + \overline{y}, \frac{\overline{xy}}{2}, \frac{1}{12} \overline{xxy} + \frac{1}{12} \overline{xyy}, \dots\right]$$

```
In[*]:= bch@5
```

Out[\*]=

$$-\frac{1}{720} \overline{xxxxy} + \frac{1}{180} \overline{xxxyy} + \frac{1}{360} \overline{xyxy} + \frac{1}{180} \overline{xyyy} + \frac{1}{120} \overline{xyxyy} - \frac{1}{720} \overline{xyyy}^2$$

```
In[*]:= bch@5 // ℓ
```

Out[\*]=

$$\begin{aligned} &-\frac{1}{720} AW[x, x, x, x, y] + \frac{1}{180} AW[x, x, x, y, x] + \frac{1}{180} AW[x, x, x, y, y] - \\ &\frac{1}{120} AW[x, x, y, x, x] - \frac{1}{120} AW[x, x, y, x, y] - \frac{1}{120} AW[x, x, y, y, x] + \\ &\frac{1}{180} AW[x, x, y, y, y] + \frac{1}{180} AW[x, y, x, x, x] - \frac{1}{120} AW[x, y, x, x, y] + \frac{1}{30} AW[x, y, x, y, x] - \\ &\frac{1}{120} AW[x, y, x, y, y] - \frac{1}{120} AW[x, y, y, x, x] - \frac{1}{120} AW[x, y, y, x, y] + \frac{1}{180} AW[x, y, y, y, x] - \\ &\frac{1}{720} AW[x, y, y, y, y] - \frac{1}{720} AW[y, x, x, x, x] + \frac{1}{180} AW[y, x, x, x, y] - \frac{1}{120} AW[y, x, x, y, x] - \\ &\frac{1}{120} AW[y, x, x, y, y] - \frac{1}{120} AW[y, x, y, x, x] + \frac{1}{30} AW[y, x, y, x, y] - \frac{1}{120} AW[y, x, y, y, x] + \\ &\frac{1}{180} AW[y, x, y, y, y] + \frac{1}{180} AW[y, y, x, x, x] - \frac{1}{120} AW[y, y, x, x, y] - \frac{1}{120} AW[y, y, x, y, x] - \\ &\frac{1}{120} AW[y, y, x, y, y] + \frac{1}{180} AW[y, y, y, x, x] + \frac{1}{180} AW[y, y, y, x, y] - \frac{1}{720} AW[y, y, y, y, x] \end{aligned}$$

```
In[*]:= τ = FreeLie`Private`τ;
```

```
In[*]:= τ[x, bch]@{6}
```

Out[\*]=

$$AS\left[AW[], -\frac{AW[y]}{2}, -\frac{1}{6} AW[x, y] + \frac{1}{12} AW[y, x] + \frac{1}{12} AW[y, y],$$

$$\begin{aligned}
 & \frac{1}{12} AW[y, x, y] - \frac{1}{24} AW[y, y, x], \frac{1}{180} AW[x, x, x, y] - \frac{1}{120} AW[x, x, y, x] - \frac{1}{120} AW[x, x, y, y] + \\
 & \frac{1}{180} AW[x, y, x, x] + \frac{1}{30} AW[x, y, x, y] - \frac{1}{120} AW[x, y, y, x] + \frac{1}{180} AW[x, y, y, y] - \\
 & \frac{1}{720} AW[y, x, x, x] - \frac{1}{120} AW[y, x, x, y] - \frac{1}{120} AW[y, x, y, x] - \frac{1}{120} AW[y, x, y, y] + \\
 & \frac{1}{180} AW[y, y, x, x] - \frac{1}{120} AW[y, y, x, y] + \frac{1}{180} AW[y, y, y, x] - \frac{1}{720} AW[y, y, y, y], \\
 & - \frac{1}{360} AW[y, x, x, x, y] + \frac{1}{240} AW[y, x, x, y, x] + \frac{1}{240} AW[y, x, x, y, y] - \\
 & \frac{1}{360} AW[y, x, y, x, x] - \frac{1}{60} AW[y, x, y, x, y] + \frac{1}{240} AW[y, x, y, y, x] - \\
 & \frac{1}{360} AW[y, x, y, y, y] + \frac{AW[y, y, x, x, x]}{1440} + \frac{1}{240} AW[y, y, x, x, y] + \frac{1}{240} AW[y, y, x, y, x] + \\
 & \frac{1}{240} AW[y, y, x, y, y] - \frac{1}{360} AW[y, y, y, x, x] - \frac{1}{360} AW[y, y, y, x, y] + \frac{AW[y, y, y, y, x]}{1440}, \\
 & - \frac{AW[x, x, x, x, x, y]}{5040} + \frac{AW[x, x, x, x, y, x]}{2016} + \frac{AW[x, x, x, x, y, y]}{2016} - \frac{AW[x, x, x, y, x, x]}{1512} - \\
 & \frac{1}{630} AW[x, x, x, y, x, y] - \frac{AW[x, x, x, y, y, x]}{5040} - \frac{AW[x, x, x, y, y, y]}{1512} + \\
 & \frac{AW[x, x, y, x, x, x]}{2016} + \frac{1}{840} AW[x, x, y, x, x, y] + \frac{1}{840} AW[x, x, y, x, y, x] + \\
 & \frac{1}{840} AW[x, x, y, x, y, y] - \frac{AW[x, x, y, y, x, x]}{5040} + \frac{1}{840} AW[x, x, y, y, x, y] - \\
 & \frac{AW[x, x, y, y, y, x]}{5040} + \frac{AW[x, x, y, y, y, y]}{2016} - \frac{AW[x, y, x, x, x, x]}{5040} - \frac{1}{630} AW[x, y, x, x, x, y] + \\
 & \frac{1}{840} AW[x, y, x, x, y, x] + \frac{1}{840} AW[x, y, x, x, y, y] - \frac{1}{630} AW[x, y, x, y, x, x] - \\
 & \frac{1}{140} AW[x, y, x, y, x, y] + \frac{1}{840} AW[x, y, x, y, y, x] - \frac{1}{630} AW[x, y, x, y, y, y] + \\
 & \frac{AW[x, y, y, x, x, x]}{2016} + \frac{1}{840} AW[x, y, y, x, x, y] + \frac{1}{840} AW[x, y, y, x, y, x] + \\
 & \frac{1}{840} AW[x, y, y, x, y, y] - \frac{AW[x, y, y, y, x, x]}{1512} - \frac{1}{630} AW[x, y, y, y, x, y] + \\
 & \frac{AW[x, y, y, y, y, x]}{2016} - \frac{AW[x, y, y, y, y, y]}{5040} + \frac{AW[y, x, x, x, x, x]}{30240} + \frac{AW[y, x, x, x, x, y]}{2016} - \\
 & \frac{AW[y, x, x, x, y, x]}{5040} - \frac{AW[y, x, x, x, y, y]}{5040} - \frac{AW[y, x, x, y, x, x]}{5040} + \frac{1}{840} AW[y, x, x, y, x, y] - \\
 & \frac{AW[y, x, x, y, y, x]}{1120} - \frac{AW[y, x, x, y, y, y]}{5040} + \frac{AW[y, x, y, x, x, x]}{2016} + \frac{1}{840} AW[y, x, y, x, x, y] + \\
 & \frac{1}{840} AW[y, x, y, x, y, x] + \frac{1}{840} AW[y, x, y, x, y, y] - \frac{AW[y, x, y, y, x, x]}{5040} + \\
 & \frac{1}{840} AW[y, x, y, y, x, y] - \frac{AW[y, x, y, y, y, x]}{5040} + \frac{AW[y, x, y, y, y, y]}{2016} - \frac{AW[y, y, x, x, x, x]}{5040} -
 \end{aligned}$$

$$\frac{AW[y, y, x, x, x, y]}{5040} - \frac{AW[y, y, x, x, y, x]}{1120} - \frac{AW[y, y, x, x, y, y]}{1120} - \frac{AW[y, y, x, y, x, x]}{5040} +$$

$$\frac{1}{840} AW[y, y, x, y, x, y] - \frac{AW[y, y, x, y, y, x]}{1120} - \frac{AW[y, y, x, y, y, y]}{5040} +$$

$$\frac{AW[y, y, y, x, x, x]}{3780} - \frac{AW[y, y, y, x, x, y]}{5040} - \frac{AW[y, y, y, x, y, x]}{5040} - \frac{AW[y, y, y, x, y, y]}{5040} +$$

$$\frac{AW[y, y, y, y, x, x]}{3780} + \frac{AW[y, y, y, y, x, y]}{2016} - \frac{AW[y, y, y, y, y, x]}{5040} + \frac{AW[y, y, y, y, y, y]}{30240}$$

```
In[*]:= {A = LS[{x, y}, As], B = LS[{x, y}, Bs]};
As[x] = 1 / 2; Bs[y] = 0;
msgs = SeriesSolve[{A, B}, h^-1 (b[LW@x, A] + b[LW@y, B] == LS[0])];
Timing[A@10;]
Length[Last[#]] & /@ Read[msgs]
```

- SeriesSolve: In degree 1 arbitrarily setting {As[x] → 0, As[y] → 0, Bs[y] → 0}.
- SeriesSolve: In degree 3 arbitrarily setting {As[x, y] → 0}.
- SeriesSolve: In degree 5 arbitrarily setting {As[x, x, y, x, y] → 0, Bs[x, x, y, y] → 0, Bs[x, x, y, y] → 0}.
- General: Further output of SeriesSolve::ArbitrarilySetting will be suppressed during this calculation.

Out[\*]=  
{0.5, Null}

Out[\*]=  
{3, 0, 1, 0, 3, 0, 6, 4, 13, 12}

```
In[*]:= Timing[A@15;]
Length[Last[#]] & /@ Read[msgs]
```

- SeriesSolve: In degree 11 arbitrarily setting {As[x, x, x, x, x, x, x, y, <<1>>] → 0, As[x, x, x, x, x, x, x, y, <<1>>] → 0, As[x, x, x, x, x, x, y, y, <<1>>] → 0, As[x, x, x, x, x, y, x, y, <<1>>] → 0, As[x, x, <<5>>, y, <<1>>] → 0, <<1>>, <<1>> → 0, As[x, x, x, x, y, x, y, <<1>>] → 0, As[x, x, x, x, y, x, y, y, <<1>>] → 0, As[x, x, x, x, y, y, y, <<1>>] → 0, <<27>>}.
- SeriesSolve: In degree 12 arbitrarily setting {As[x, x, x, x, x, x, x, y, <<2>>] → 0, As[x, x, x, x, x, x, y, <<2>>] → 0, As[x, x, x, x, x, y, x, y, <<2>>] → 0, As[x, x, <<5>>, x, y, <<2>>] → 0, <<1>>, <<1>> → 0, As[x, x, x, x, y, x, y, <<2>>] → 0, As[x, x, x, x, y, x, y, y, <<2>>] → 0, As[x, x, x, x, y, y, y, <<2>>] → 0, <<30>>}.
- SeriesSolve: In degree 13 arbitrarily setting {As[x, x, x, x, x, x, x, x, <<3>>] → 0, As[x, x, x, x, x, x, x, x, <<3>>] → 0, As[x, x, x, x, x, x, x, y, <<3>>] → 0, As[x, x, x, x, x, x, y, x, <<3>>] → 0, As[x, x, <<5>>, y, x, <<3>>] → 0, <<1>>, <<1>> → 0, As[x, x, x, x, x, x, y, y, <<3>>] → 0, As[x, x, x, x, x, y, x, y, <<3>>] → 0, As[x, x, x, x, x, y, y, <<3>>] → 0, <<89>>}.
- General: Further output of SeriesSolve::ArbitrarilySetting will be suppressed during this calculation.

Out[\*]=  
{197.516, Null}

Out[\*]=  
{3, 0, 1, 0, 3, 0, 6, 4, 13, 12, 37, 40, 99, 140, 284}

