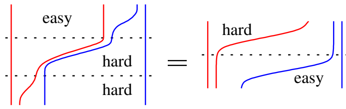
See also Journal, BBS: Kuno.

230404 **Q.** Is the pentagon in emergent 2-poles 2-strands equivalent to the standard FL-pentagon? (The spaces grow slower!) Can Φ_{pps} be found degree by degree? Is there a GT group? Isomorphic to a known variant?



In 240510_toward_KV1.pdf:

The emergent linearized ⬠:

1. $\varphi(x+y,0)-\varphi(y,0)=0$.
2. With $R=R(\varphi)$, $R(x+y,0)-R(y,0)=0$.
3. $(\partial_y\varphi)(x,y)+(\partial_y\varphi)(y,0)-((\partial_y\varphi)(x+y,0)=2R(x,y)$.

$R\colon FL(x,y)\to FA(x,y)$ satisfies $R(x)=R(y)=0$ and

$$R([u,v])=[u,R(v)]+[R(u),v] + \frac{1}{2}\begin{pmatrix}(\partial_x v)x(\partial_x u)^*-(\partial_x u)x(\partial_x v)^*\\ +(\partial_y v)y(\partial_y u)^*-(\partial_y u)y(\partial_y v)^*\end{pmatrix}.$$

Lemma 1. $\forall m\geqslant 0$, $\partial_x(\mathrm{ad}_x^m(y))=-\sum_{i+j=m-1}x^i\,\mathrm{ad}_x^j(y)$.

Lemma 2. $\partial_x(\mathrm{ad}_x^m(y))=-\sum_{b+c=m-1}(-1)^c\binom{m}{b}x^byx^c$.

Lemma 3. $R(\mathrm{ad}_x^m(y))=\sum_{i+j=m-1}\frac{(-1)^j}{2}\left(\binom{m-1}{j-1}-\binom{m-1}{i-1}\right)x^iyx^j$.

Lemma 4. If $\varphi\in$ SolEmPent is degree m where $m\geqslant 4$ is even then $\varphi_1=0$, where φ_j is the y-degree j part of φ.

Lemma 5. If $\varphi\in$ SolEmPent is of degree $\geqslant 3$, then $(\partial_y\varphi)^*=\partial_y\varphi$.

Theorem 6. (Schneps, *Double Shuffle and Kashiwara-Vergne Lie Algebras*). Let $b\in FL(x,y)$. Then $\exists a\in FL(x,y)$ s.t. $[x,a]+[y,b]=0$ iff $\partial_y b=\partial^y b$ $(=(\partial_y b)^*)$.

Proof. ? □

Claim 7. If $\varphi\in$ SolEmPent is of degree $\geqslant 3$ and if we learn that $(\partial_x\varphi)^*=(\partial_x\varphi)(y,x)$, then $\nu(\varphi):=(\varphi(y,x),\varphi(x,y))$ satisfies KV1.

In 240520_EM5-gon_and_KV2.pdf:

$\eta\colon \mathbb{Q}\pi\otimes\mathbb{Q}\pi\to\mathbb{Q}\pi$: The homotopy intersection form, $\eta(\alpha,\beta):=\sum_{p\in\alpha\cap\beta}(-)^p\alpha_{*p}\beta_{p*}$.

$\mu_0\colon\mathbb{Q}\pi\to\mathbb{Q}\pi$: Self intersect and prune: $\mu_0(\gamma):=\sum_{p\in\gamma\cap\gamma}(-)^p\gamma_{*p_1}\gamma_{p_2*}$. Satisfies $\mu_0(\alpha\beta)=\mu_0(\alpha)\beta+\alpha\mu_0(\beta)+\eta(\alpha,\beta)$. Recovers the Turaev δ!

Theorem 8. (Massuyeau-Turaev, Naef). For $\varphi\in\mathrm{tAut}(FL(z_1,\dots,z_n)$, $\varphi/\!\!/\eta_a=\eta_a\otimes\eta_a/\!\!/\varphi$ iff $\varphi(\sum z_i)=\sum z_i$.

Proof. ? □

Lemma 9. Let $u=(u_1,\dots,u_n)\in\mathfrak{tder}$. Then $u(\sum z_i)=0$ iff $\forall i,j\;\partial_j u_i=\partial^i u_j$.

Proof. $\Rightarrow$ is easy using the Drinfel'd lemma. □

Proposition 10. $R=-\frac{1}{2}\mu_{0a}$.

In 240605_symmKV_2_EM5-gon:

Theorem 11. (AKKN). With $\mathfrak{krv}_n^0:=\{\tilde{u}\in\mathfrak{krv}_n:\mathrm{div}(\tilde{u})\in\oplus_i\mathbb{Q}|z_i|\}$,

$$\tilde{u}\in\mathfrak{krv}_n^0\Leftrightarrow(\tilde{u}\circ\eta_a=\eta_a\circ\tilde{u})\wedge(\tilde{u}\circ\mu_{0a}=\mu_{0a}\circ\tilde{u}).$$

Theorem 12. If $\varphi\in FL(x,y)$ has $\deg\varphi\geqslant 3$ and $(\varphi(y,x),\varphi(x,y))\in\mathfrak{krv}_2^{\mathrm{sym}}$, then $\varphi\in$ SolEmPent.