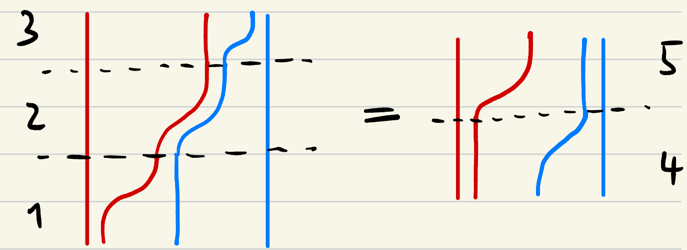
(linearlized) pentagon eq. in emergent $\mathcal{P}$

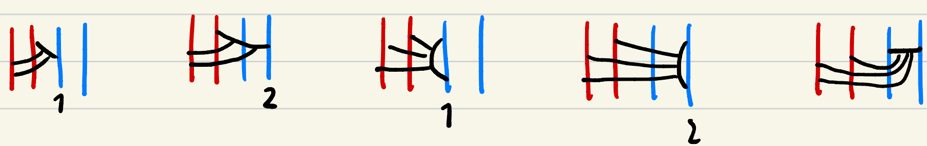
$$\varphi \in FL\langle x,y\rangle \overset{\text{hor}}{\subset} A(||||) \supset \mathcal{P} = FL \oplus FA[1]$$

$$\varphi \longmapsto \psi + R$$

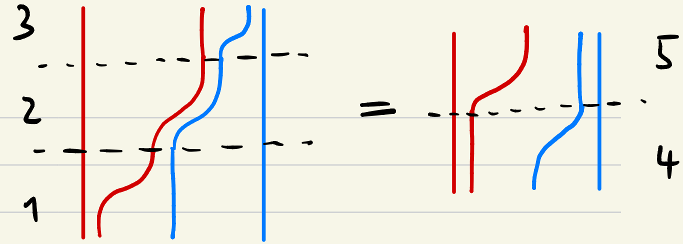
$$\boxed{\varphi^1+\varphi^2+\varphi^3=\varphi^4+\varphi^5} \in \mathcal{P}(||||)$$



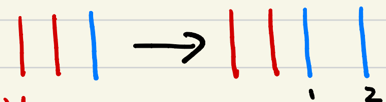
$$\mathcal{P}(||||) = FL \oplus FL \oplus FA \oplus FA \oplus FA$$



$$\varphi = \underset{U}{\cancel{\Psi}} + R$$

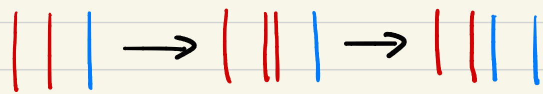


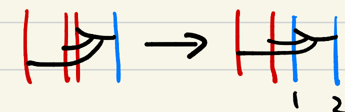
$\boxed{\varphi^1}$



$$U\ \cancel{\Psi} = \overset{U}{\cancel{\Psi}}(x,y) \longmapsto \overset{U}{\cancel{\Psi}}_1(x,y)\ ,\quad R = R(x,y) \longmapsto R_1(x,y)$$

$\boxed{\varphi^2}$



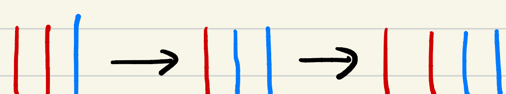


$$U = U(x,y) \longmapsto U(x,y+z) \longmapsto (\partial_y U)_{12}(x,y) + U_2(x,y)$$

$$V(x,y,z) \longmapsto (\partial_z V)_{12}(x,y,0) + V_2(x,y,0)$$

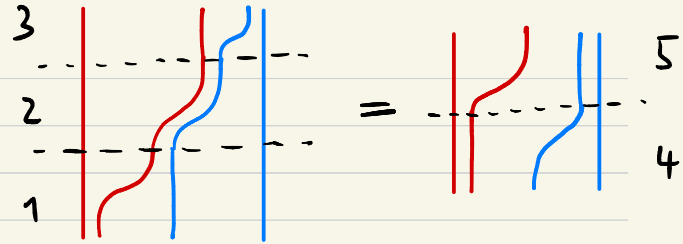
$$R = R(x,y) \longmapsto R(x,y+z) \longmapsto R_2(x,y)$$

$\boxed{\varphi^3}$

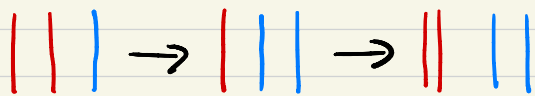


$$U = U(x,y) \longmapsto (\partial_y U)_{12}(x,0) + U_2(x,0) \longmapsto (\partial_y U)_{12}(y,0) + U_2(y,0)$$

$$R = R(x,y) \longmapsto R_2(x,0) \longmapsto R_2(y,0)$$

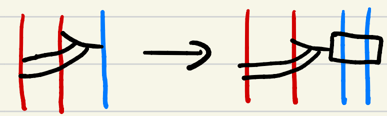


$\boxed{\varphi^4}$

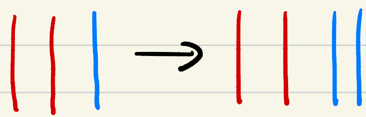


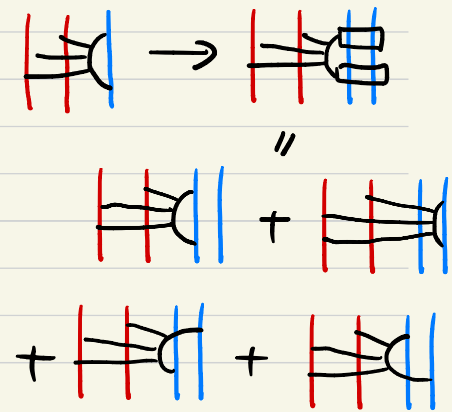
$$u = u(x,y) \mapsto (\partial_y u)_{12}(x+y,0) + u_2(x+y,0)$$

$$R = R(x,y) \mapsto R_2(x+y,0)$$



$\boxed{\varphi^5}$





$$u = u(x,y) \mapsto u_1(x,y) + u_2(x,y)$$

$$R = R(x,y) \mapsto R_1(x,y) + R_2(x,y)$$
$$\pm \left(R_{12}(x,y) + (R^*)_{12}(x,y) \right)$$

$$y = U + R$$

$$\rightsquigarrow \quad \overset{1}{\cancel{U_1(x,y)}} + \cancel{R_1(x,y)} + \Big((\partial_y U)_{12}(x,y) + \overset{2}{\cancel{U_2(x,y)}} + \cancel{R_2(x,y)} \Big)$$

$$+ \Big((\partial_y U)_{12}(y,0) + \overset{3}{U_2(y,0)} + R_2(y,0) \Big)$$

$$= (\partial_y U)_{12}(x+y,0) + \overset{4}{U_2(x+y,0)} + R_2(x+y,0)$$

$$+ \cancel{U_1(x,y)} + \cancel{U_2(x,y)} + \cancel{R_1(x,y)} + \cancel{R_2(x,y)} \overset{5}{\pm} \Big(R_{12}(x,y) + (R^*)_{12}(x,y) \Big)$$

$$0 = \underbrace{U_2(y,0) - U_2(x+y,0)}_{FL_2} + \underbrace{R_2(y,0) - R_2(x+y,0)}_{FA_2}$$

$$+ \underbrace{(\partial_y U)_{12}(x,y) + (\partial_y U)_{12}(y,0) - (\partial_y U)_{12}(x+y,0) \mp \underbrace{\Big(R_{12}(x,y) + (R^*)_{12}(x,y) \Big)}_{= 2R_{12}}}_{FA_{12}}$$

$R = R(u)$

? ...

$$\underbrace{u_2(y,0) - u_2(x+y,0)}_{FL_2} + \underbrace{R_2(y,0) - R_2(x+y,0)}_{FA_2}$$

$$\underbrace{+ (\partial_y u)_{12}(x,y) + (\partial_y u)_{12}(y,0) - (\partial_y u)_{12}(x+y,0) \mp \underbrace{\big(R_{12}(x,y) + (R^*)_{12}(x,y)\big)}_{=2R_{12}\ ?}}_{FA_{12}}$$

$R = R(u)$

σ_{2n+1}

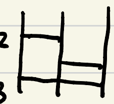
$\sigma_3, \sigma_5, \sigma_7, \sigma_9, \sigma_{11}$

$[\sigma_3, [\sigma_3, \sigma_5]]$

$u + R$

original

$\Phi \in \exp(\mathfrak{t}_3)$

t_{12} t_{13}  t_{23}

$t_{12} + t_{23} + t_{13} = 0$

$\Phi = \Phi(x,y) \in \exp FL\langle x, y\rangle$

$\mathrm{grt}_1 \longrightarrow \mathrm{kv}$

$\psi \longmapsto \big(\psi(-x-y, x), \psi(-x-y, y)\big)$