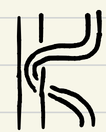
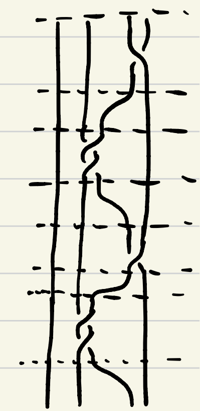
$$\Delta_{1\to 1,2} : \mathcal{A}\Big(\underset{x\,y\,1}{|\,|\,|}\Big) \longrightarrow \mathcal{A}\Big(\underset{x\,y\,1\,2}{|\,|\,|\,|}\Big),$$

$$U = U(x,y) \mapsto \mathbb{O}_{12}\Big(U'(x_1,y_1)\,U''(x_2,y_2) + t\,\mathcal{D}(U)\Big)$$

$$\mathcal{D}(U) = (1\otimes\partial_x)\,\partial_x(U) \;/\!/\; \cdots + (1\otimes\partial_y)\,\partial_y(U) \;/\!/\; \cdots$$

6-gon for β: $\quad \Delta_{1\to 1,2}(Z(\beta)) = Z(\Delta_{1\to 1,2}(\beta))$

$$Z(\beta) = \Phi(x,y)\,e^{y}\,\Phi(x,y)^{-1} =: B(x,y)$$

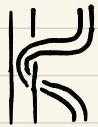
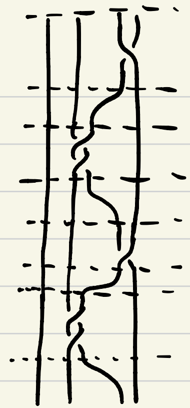
$\Delta_{1\to 1,2}(\beta) =$  $=$ 

Assume:

$\bar{\Phi} := \Phi \;/\!/\; \underset{x\,y\,1}{|\,|\,|} \rightsquigarrow \underset{x\,1\,2}{|\,|\,|}$ is a pss associator.

$\rightsquigarrow \bar{\Phi} = 1 + t^{12}\,h(x_1 - \bar{x}_1)$ with h^{odd}: Bernoulli

$$Z(\beta) = \Phi e^{y} \Phi^{-1}$$

$$\Delta_{1\to 1,2}(\beta) =$$  $=$ 

$$\xrightarrow{Z} 1 + \tfrac{1}{2} t^{12} \quad (\text{with } \| \times \;)$$

$$\Delta_{x \to x,y}\big(h(x_1 - \bar{x}_1)\big) = h$$

$$\xrightarrow{Z} (1 - t^{12} h)\, B(x_1, y_1)\, (1 + t^{12} h)$$

$$= \mathbb{D}_{12}\Big(B(x_1,y_1) + t^{12}\big(B(x_1,y_1) - B(\bar{x}_1,\bar{y}_1)\big) h\Big)$$

.....

$$Z\big(\Delta_{1\to 1,2}(\beta)\big) = \mathbb{D}_{12}\Bigg(B(x_1,y_1) B(x_2,y_2) + t^{12}\Big(\tfrac{1}{2} B(x_1,y_1)\big(B(\bar{x}_2,\bar{y}_2) - B(x_2,y_2)\big) + B(x_1,y_1)\big(B(x_2,y_2) - B(\bar{x}_2,\bar{y}_2)\big) h(x_2 - \bar{x}_2 + y_2 - \bar{y}_2) + \big(B(x_1,y_1) - B(\bar{x}_1,\bar{y}_1)\big) B(\bar{x}_2,\bar{y}_2)\, h(x_1 - \bar{x}_1 + y_1 - \bar{y}_1) \Big)\Bigg)$$

$$\Delta_{1\to 1.2}\big(Z(\beta)\big) = \mathbb{D}_{12}\Big(B'(x_1,y_1) B''(x_2,y_2) + t^{12} \mathfrak{D}(B)\Big)$$

looks complicated gets simpler under HOMFLY-PT ?

6-gon for β : $\mathcal{D}(B) = \cdots\cdots$

$$B = \Phi e^{y} \Phi^{-1}$$

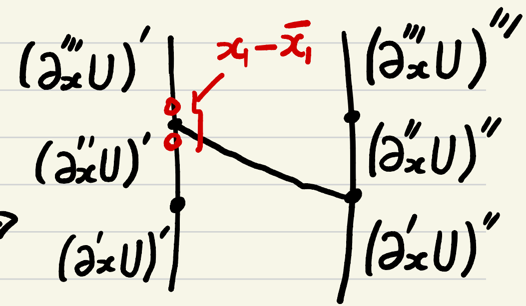
5-gon : $\mathcal{D}(\Phi) = \cdots\cdots$

$\mathcal{D}(\Phi e^{y} \Phi^{-1}) = \cdots$ in terms of $\mathcal{D}(\Phi)$

$$\mathcal{D} : \mathcal{A}(\underset{x\ y\ 1}{|||}) \longrightarrow \mathcal{A}(\underset{x\ y\ 1\ 2}{||\ \text{H}})$$

$\Downarrow$

$$U = U(x,y) \longmapsto (\partial'_x U) \otimes (\partial''_x U) \otimes (\partial'''_x U) \xmapsto{\Delta^{\otimes 3}}$$



$\mathcal{D}([a,b])$ + (Similar term with $x \leftrightarrow y$)

$\mathcal{D}$