

Pensieve header: A fast Jones program for Stavros. Based on pensieve://Classes/23-FastComputations/.

```
In[*]:= << KnotTheory`
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[*]:= PK = PD[
```

```

  X[4, 48, 5, 47], X[5, 40, 6, 41], X[6, 68, 7, 67], X[11, 97, 12, 96],
  X[12, 62, 13, 61], X[13, 26, 14, 27], X[14, 82, 15, 81], X[20, 105, 21, 106],
  X[22, 43, 23, 44], X[23, 65, 24, 64], X[24, 9, 25, 10], X[25, 99, 26, 98],
  X[29, 18, 30, 19], X[30, 90, 31, 89], X[31, 2, 32, 3], X[32, 76, 33, 75],
  X[35, 53, 36, 52], X[36, 72, 37, 71], X[39, 49, 40, 48], X[44, 21, 45, 22],
  X[45, 87, 46, 86], X[50, 38, 51, 37], X[51, 70, 52, 71], X[54, 34, 55, 33],
  X[55, 74, 56, 75], X[56, 4, 57, 3], X[57, 88, 58, 89], X[58, 20, 59, 19],
  X[62, 97, 63, 98], X[63, 11, 64, 10], X[68, 49, 69, 50], X[69, 39, 70, 38],
  X[72, 53, 73, 54], X[73, 35, 74, 34], X[76, 2, 77, 1], X[77, 90, 78, 91],
  X[78, 18, 79, 17], X[82, 99, 83, 100], X[83, 9, 84, 8], X[84, 65, 85, 66],
  X[85, 43, 86, 42], X[87, 105, 88, 104], X[92, 16, 93, 15], X[93, 80, 94, 81],
  X[94, 28, 95, 27], X[95, 60, 96, 61], X[100, 8, 101, 7], X[101, 66, 102, 67],
  X[102, 42, 103, 41], X[103, 46, 104, 47], X[106, 60, 107, 59], X[107, 28, 108, 29],
  X[108, 80, 109, 79], X[109, 16, 110, 17], X[110, 92, 1, 91]
];
```

```
In[*]:= J1[pd_] := Module[{w, p, t1, t2, t3, t4, t5},
```

```

  w = Plus @@ (pd /. {
    X[i_, j_, k_, l_] /; j - l == 1 ∨ l - j > 1 => 1,
    X[i_, j_, k_, l_] /; l - j == 1 ∨ j - l > 1 => -1
  });
```

```
SetAttributes[p, Orderless];
```

```
t1 = pd /. X[i_, j_, k_, l_] => A p[i, j] p[k, l] + B p[i, l] p[j, k];
```

```
t2 = Expand[t1 /. PD -> Times];
```

```
t3 = t2 //. p[i_, j_] p[j_, k_] => p[i, k];
```

```
t4 = t3 /. {p[i_, i_] => d, p[i_, j_]^2 => d};
```

```
t5 = Expand[t4 /. {B -> 1 / A, d -> -A^2 - A^-2}];
```

```
Simplify[(-A^3)^-w  $\frac{t5}{-A^2 - A^{-2}}$  /. A -> q^-1/4]
```

```
]
```

```

In[*]:= J2[pd_] := Module[{w, front, TL, todo, v, x, t1, t2, t3, t4, B, d},
  w = Plus @@ (pd /. {
    X[i_, j_, k_, l_] /; j - l == 1 ∨ l - j > 1 ∴ 1,
    X[i_, j_, k_, l_] /; l - j == 1 ∨ j - l > 1 ∴ -1
  });
  SetAttributes[p, Orderless];
  front = {};
  TL = 1;
  todo = List @@ pd;
  v[x_] := Length[front ∩ List @@ x];
  While[Length[todo] > 0,
    x = RandomChoice[MaximalBy[todo, v]];
    t1 = TL (x /. X[i_, j_, k_, l_] ∴ A p[i, j] p[k, l] + B p[i, l] p[j, k]);
    t2 = Expand[t1];
    t3 = t2 /. p[i_, j_] p[j_, k_] ∴ p[i, k];
    t4 = t3 /. {p[i_, i_] ∴ d, p[i_, j_]² ∴ d};
    TL = Expand[t4 /. {B → 1 / A, d → -A² - A⁻²}];
    todo = Complement[todo, {x}];
    front = Complement[front ∪ List @@ x, front ∩ List @@ x];
  ];
  Simplify[ $(-A^3)^{-w} \frac{TL}{-A^2 - A^{-2}}$  /. A → q-1/4]
]

```

In[*]:= Timing@J2[PK]

Out[*]=

$$\left\{ 4.17188, -\frac{1 - 3q + q^2 + q^3 - 2q^4 + q^5 + q^7 - q^8 + q^{10} - q^{11} + q^{13} - q^{14} - q^{15} + q^{16} - q^{18} + q^{20} + q^{25} - q^{26}}{q^2} \right\}$$