This is my thesis.
And here is the code:

```
g_1[h[F_, L_]] := h[
  Permute[F, Cycles[{{1, 2}}]],
  Expand[
   L /. Join[{x_2 -> -t_{F[[1]]} x_2},
     Table[x_j -> (1 - t_{F[[j]]})/(t_{F[[2]]} - 1) x_2 + (t_{F[[1]]} - 1)/(t_{F[[2]]} - 1) x_j, {j, 3, Length[F]}]]] //
    Simplify
  ]
ḡ_1[h[F_, L_]] := h[
  Permute[F, Cycles[{{1, 2}}]],
  Expand[
   L /. Join[{x_2 -> -1/t_{F[[2]]} x_2},
     Table[x_j -> (1 - t_{F[[j]]})/(t_{F[[2]]} (t_{F[[2]]} - 1)) x_2 + (t_{F[[1]]} - 1)/(t_{F[[2]]} - 1) x_j, {j, 3, Length[F]}]]] //
    Simplify
  ]
g_i_[h[F_, L_]] /; i > 1 := h[
  Permute[F, Cycles[{{i, i + 1}}]],
  Expand[L /. {x_i -> x_{i+1}, x_{i+1} -> t_{F[[i]]} x_i + (1 - t_{F[[i+1]]}) x_{i+1}}] // Simplify
 ]
ḡ_i_[h[F_, L_]] /; i > 1 := h[
  Permute[F, Cycles[{{i, i + 1}}]],
  Expand[L /. {x_i -> 1/t_{F[[i+1]]} x_{i+1} + (t_{F[[i]]} - 1)/t_{F[[i+1]]} x_i, x_{i+1} -> x_i}] // Simplify
  ]
```

```
ι[h[F_, L1_], h[F_, L2_]] :=
 Simplify[Expand[L1 (L2 /. {t_i_ :> t_i^-1, x_i_ :> x̄_i})] /.
```

$$\left\{x_{i\_}\,\overline{x}_{j\_} :\!\to \begin{cases} \dfrac{(t_{F[[1]]}-1)\,(t_{F[[i]]}-1)\,(1-t_{F[[1]]}\,t_{F[[i]]})}{t_{F[[1]]}\,t_{F[[i]]}} & i == j \\[2ex] \dfrac{-(t_{F[[1]]}-1)\,(t_{F[[i]]}-1)\,(t_{F[[j]]}-1)}{t_{F[[j]]}} & i < j \\[2ex] \dfrac{-(t_{F[[1]]}-1)\,(t_{F[[i]]}-1)\,(t_{F[[j]]}-1)}{t_{F[[1]]}\,t_{F[[j]]}} & i > j \end{cases}\right\}\Bigg]$$

Above was the program. Let's test it:

```
Table[ι[h[{1, 2, 3, 4}, x_i], h[{1, 2, 3, 4}, x_j]], {i, 2, 4}, {j, 2, 4}] //
 MatrixForm
```

$$\begin{pmatrix} \frac{(-1+t_1)\,(-1+t_2)\,(1-t_1\,t_2)}{t_1\,t_2} & -\frac{(-1+t_1)\,(-1+t_2)\,(-1+t_3)}{t_3} & -\frac{(-1+t_1)\,(-1+t_2)\,(-1+t_4)}{t_4} \\ -\frac{(-1+t_1)\,(-1+t_2)\,(-1+t_3)}{t_1\,t_2} & \frac{(-1+t_1)\,(-1+t_3)\,(1-t_1\,t_3)}{t_1\,t_3} & -\frac{(-1+t_1)\,(-1+t_3)\,(-1+t_4)}{t_4} \\ -\frac{(-1+t_1)\,(-1+t_2)\,(-1+t_4)}{t_1\,t_2} & -\frac{(-1+t_1)\,(-1+t_3)\,(-1+t_4)}{t_1\,t_3} & \frac{(-1+t_1)\,(-1+t_4)\,(1-t_1\,t_4)}{t_1\,t_4} \end{pmatrix}$$

And now let's move on to the next chapter.