This is just a test.

$$\int_{-\infty}^{\infty} e^{-x^2/2}\,dx = \sqrt{2\pi}$$

Now the CPU is less loaded.

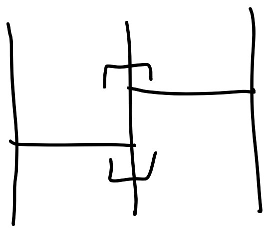
Perhaps it is acceptable

This is just a test

What if I write slowly?

This is a test once again.

$t = (x_1 - x_2)(p_1 - p_2)$ $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{pmatrix}$

 = ... Seems to satisfy 4T!

$$[(x_1 - x_2)(p_1 - p_2), (x_2 - x_3)(p_2 - p_3)] =$$

Do I know this weight system?

$y \longrightarrow p$

$b \longrightarrow c$

$a \longrightarrow pq$

$x \longrightarrow q$

$$yx + xy + ab + ba$$

$$=$$

Which condition on DPG_n is better?

1. No $\text{greek}_0\,\text{greek}_n$ terms,
   ↑ weights ↑

oops, there is such a term in ρ

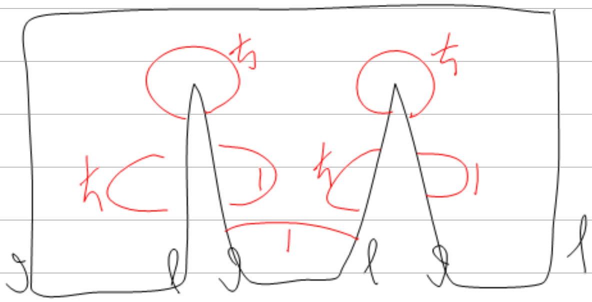
No "$\beta\alpha$" – $[b,a]=0$
"weight-0 latin variables are central and primitive"

or

2. The coefficient of any wt-n variable must be at most linear in the wt-0 variables.

at $\epsilon=0$

No bba



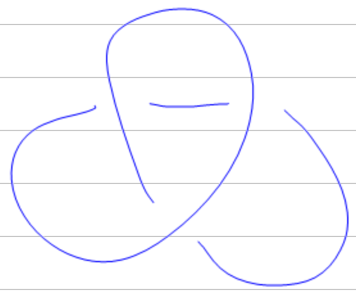
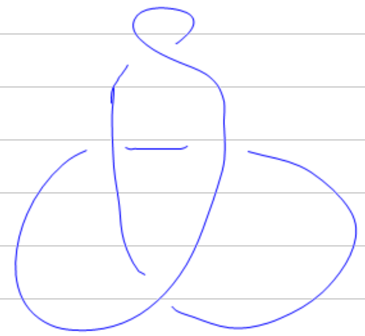
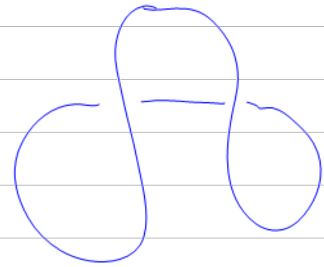
$\partial_\lambda W^j = W^i \partial_{z^i} W^j$

"The wake equation"?

What constant-λ equations is W required to satisfy?

$$\partial_\lambda Z = \frac{1}{2} \sum F_{ij} \left(\partial_{ij} Z + (\partial_i Z)(\partial_j Z) \right)$$

$$\partial_\lambda Z = \mathcal{F} Z$$

Aug 14, 2020

$\bar{g} = g - 1, \quad g = \bar{g} + 1, \quad \overline{g^{-1}} = -\bar{g} g^{-1}, \quad g^{-1} = 1 - \bar{g} g^{-1}$

$g_i \bar{g}_j = (\bar{g}_i + 1)(g_j - 1) = \bar{g}_i g_j - \bar{g}_i + \bar{g}_j$

August 31, 2020

The co-invariants of ybax @ $\epsilon=0$

$[x,y]=\underset{\text{central}}{b}$, $[a,x]=x$ $[a,y]=-y$

Kill ba?. $[y,ax]=yx-ba$

$[x,ya]=ba-yx$

$$[y,x^n y^{n-1}]=nbx^{n-1}y^{n-1}$$

$[y,ax^2y]=$
$=x^2y^2+2baxy$

imbalanced is dead. using $[a,-]$

bx^ny^n is dead using $[y,x^{n+1}y^n]$

$b^{k+1}a^k x^n y^n$ is dead for $n\geq 1$,

using $[y,b^k a^k x^n y^{n-1}]\sim b^k a^{k-1}\cdot k x^n y^n$ + more bs
$+nb^{k+1}a^k x^{n-1}y^{n-1}$

needs details.

$b^{k+1}a^k x^n y^n$ is dead using induction on k and
$[x,b^k a^k x^n y^{n+1}]\sim b^k a^{k-1}x^{n+1}y^{n+1}\cdot k$
$+b^{k+1}a^k x^n y^n\cdot(n+1)$

~~Conjecture. The co-invariants are generated by~~
$\cancel{\{(ba)^k a^l x^n y^m\}}$

$[a,x]=x \iff ax=x(a+1)$
$\implies f(a)x=xf(a+1)$
$\implies [f(a),x]=x(f(a+1)-f(a))$

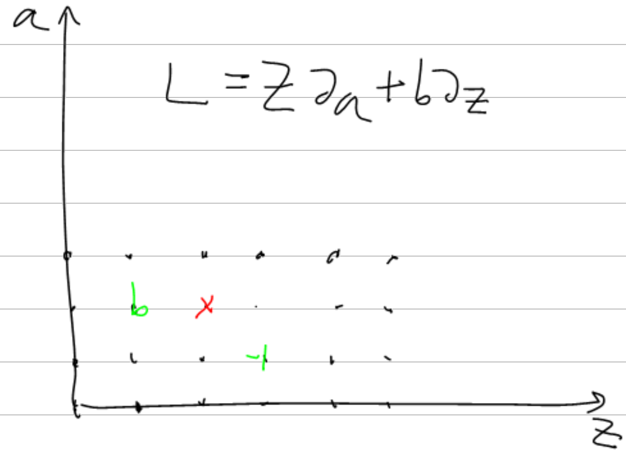
Very loosely, $L=ad_x=z\partial_a+b\partial_z$, $z=xy$ in $\{b,a,z\}$

Can kill $\{bz^n\}$ as $Lz^{n+1}=(n+1)bz^n$

Can kill $b(ba)^k z^n$ as $L(ba)^k z^{n+1}$

induct on k. $=kb(ba)^{k-1}z^{n+2}+(n+1)b(ba)^k z^n$

Conjecture: The co-kernel of L is not freely generated by $\{(ba)^k a^l z^n\}$



$L = z\partial_a + b\partial_z$

So $\mathrm{coker}(L)$ is freely generated by the first column: $\{a^n\}$

$[x,a] = -x \Rightarrow xa = (a-1)x \Rightarrow xa^k = (a-1)^k x \Rightarrow [x,a^k] = ((a-1)^k - a^k)x$

$\Rightarrow [x, \binom{a+k}{k}] = \left(\binom{a+k-1}{k} - \binom{a+k}{k}\right)x = -\binom{a+k-1}{k-1}x$

also, $[a,y] = -y \Rightarrow ay = y(a-1) \Rightarrow [\binom{a+k}{k}, y] = -y\binom{a+k-1}{k-1}$

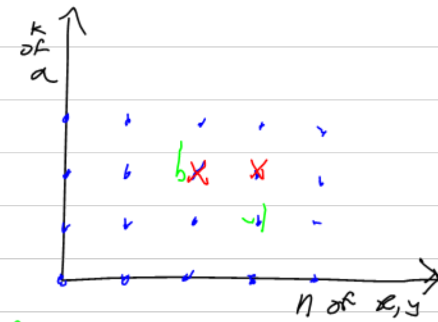
Back to $ybax_0$:

$$[x, y^{n+1}\binom{a+k}{k}x^n] = (n+1)by^n\binom{a+k}{k}x^n - y^{n+1}\binom{a+k-1}{k-1}x^{n+1}$$

$\Longrightarrow$ in co-invts,

$$y^n\binom{a+k}{k}x^n = nby^{n-1}\binom{a+k+1}{k+1}x^{n-1} = \cdots = n!\,b^n\binom{a+k+n}{k+n}$$

Edge issues ignored!



$L\left(\sum \frac{\zeta^n z^n}{n!}\right) =$

$z_n \underset{B}{\overset{A}{\rightleftarrows}} \binom{z+n}{n}$

$$e^{\zeta z} = \sum \overbrace{(e^{-\zeta}-1)}^{?}{}^n \binom{z+n}{n}$$

$$\| $$

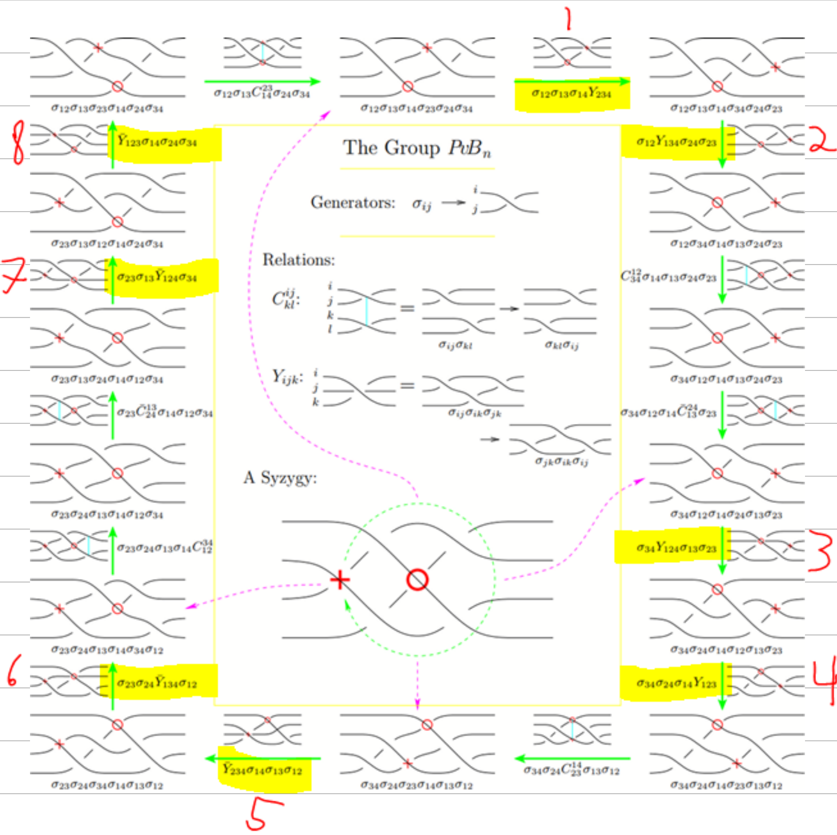
$$(e^{\zeta})^z = (1-(1-e^{-\zeta}))^{-z} = \cdots$$

Aside: $\sum x^n \binom{z+n}{n} =$

Aside: $\sum_{m \le n} \frac{\alpha^n a^n}{n!} \frac{(\beta b)^m}{m!} = ?$

Aside: $\displaystyle\sum_{m\leq n} (\alpha a)^n \frac{(\beta b)^m}{m!} = \sum_{m} \frac{(\beta b)^m}{m!} (\alpha a)^m \frac{1}{1-\alpha a}$

$$\sum_{m\leq n} \frac{(\alpha a)^n}{n!} (\beta b)^m = \sum_{n\geq 0} \frac{(\alpha a)^n}{n!} \frac{(\beta b)^n - 1}{\beta b - 1}$$



1 $\sigma_{12}\sigma_{13}\sigma_{14}Y_{234}$

2

3

4

5

6

7

8

Baby DoPeGDO Notes

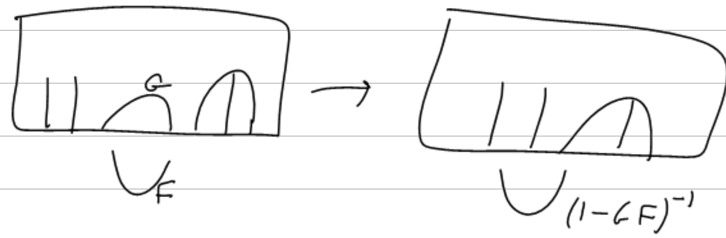
FZip calls Zip calls PZip

Fast — universal, straightforward — perturbation-only

Perhaps better,

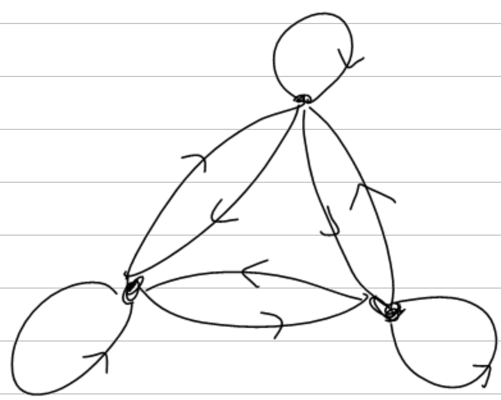
Zip calls FPZip calls

clear Gaussing, then call FPZip — PZip straightforward perturbation-only — Encapsulate then call PZip

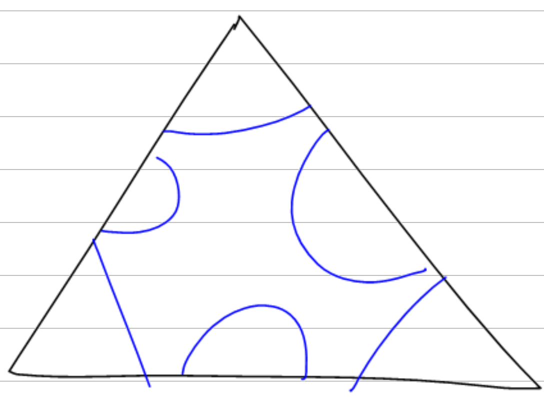


$$\text{Zip} \longrightarrow \text{PZip}$$
$$\text{FZip} \longrightarrow \text{FPZip} \longrightarrow \text{PZip}$$

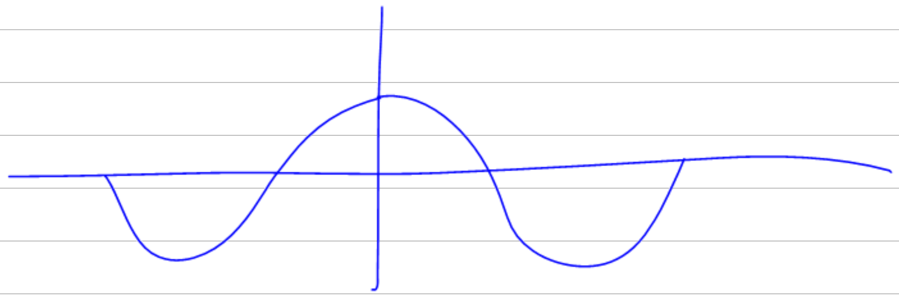
$\bar{a} := a/w \qquad \bar{a}\cdot\bar{b} = \overline{ab/w}$



$[1,2]^* = 2^*(12^*)^*$



$\exists$? Cont. $f:\mathbb{R}\to\mathbb{R}$ s.t. $f\circ f=\cos$?



$$I_{v \to w} := K_V(P_v B \longrightarrow P_w B)$$

¿ Matrices can be inverted using a restricted type of elementary ops, which preserve some divisibility condition, which prevents blowups ?

$$W : \begin{pmatrix} \Xi & \phi \\ \theta & \alpha \end{pmatrix} \longmapsto \frac{1}{\alpha}\begin{pmatrix} 1 & -\theta \\ \phi & \alpha\Xi - \phi\theta \end{pmatrix}$$

$$\frac{1}{\lambda}\begin{pmatrix} \Xi & \phi \\ \theta & \alpha \end{pmatrix} \longmapsto \frac{1}{\alpha/\lambda}\begin{pmatrix} 1 & -\theta/\lambda \\ \phi/\lambda & \frac{1}{\lambda^2}(\alpha\Xi - \phi\theta) \end{pmatrix}$$

$$= \frac{1}{\alpha\lambda}\begin{pmatrix} \lambda^2 & -\lambda\theta \\ \lambda\phi & \alpha\Xi - \phi\theta \end{pmatrix}$$

$$\begin{pmatrix} 1/\alpha & & & & & \\ -\beta/\alpha & 1 & & & & \\ -\gamma/\alpha & 0 & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{pmatrix} \qquad \frac{\epsilon}{\alpha} - \frac{\gamma}{\beta}\frac{\delta}{\alpha}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{\beta}{\alpha} & \frac{\delta}{\alpha} & & \\ 0 & \frac{\gamma}{\alpha} & \frac{\epsilon}{\alpha} & & \\ 0 & & & & \\ 0 & & & & \end{pmatrix}$$

181222 If $A \in M_{n\times n}$ and $\omega = |A|$, then each entry of A^{-1} has denominator ω, so expect $|A^{-1}| \propto \omega^{-n}$. Yet $|A^{-1}| = \omega^{-1}$. Likewise for $A := \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & b\bar{d} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a - b\bar{d}c & 0 \\ 0 & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \bar{d}c & 1 \end{pmatrix}$ and $|A| = |d||a - b\bar{d}c|$, with $\bar{d} := d^{-1}$. 2019-03, →p3:**190312**. Also,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \begin{pmatrix} (a - b\bar{d}c)^{-1} & -(a - b\bar{d}c)^{-1}b\bar{d} \\ -\bar{d}c(a - b\bar{d}c)^{-1} & \bar{d} + \bar{d}c(a - b\bar{d}c)^{-1}b\bar{d} \end{pmatrix}.$$

181222 If $A \in M_{n\times n}$ and $\omega = |A|$, then each entry of A^{-1} has denominator ω, so expect $|A^{-1}| \propto \omega^{-n}$. Yet $|A^{-1}| = \omega^{-1}$. Likewise for $A := \begin{pmatrix} {}_p a_p & {}_p b_q \\ {}_q c_p & {}_q d_q \end{pmatrix} = \begin{pmatrix} 1 & b\bar d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a - b\bar d c & 0 \\ 0 & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \bar d c & 1 \end{pmatrix}$ and $|A| = |d||a - b\bar d c|$, with $\bar d := d^{-1}$. 2019-03, →p3:**190312**. Also,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \begin{pmatrix} (a-b\bar d c)^{-1} & -(a-b\bar d c)^{-1} b\bar d \\ -\bar d c(a-b\bar d c)^{-1} & \bar d + \bar d c(a-b\bar d c)^{-1} b\bar d \end{pmatrix}.$$

$$a(a-b\bar d c)^{-1} - \overline{b\bar d c}(a-b\bar d c)^{-1} = 1$$

$$-a(a-b\bar d c)^{-1} b\bar d + \underline{b\bar d} + b\bar d c(a-b\bar d c)^{-1} b\bar d = (1 + (-a + b\bar d c)(a-b\bar d c)^{-1}) b\bar d$$

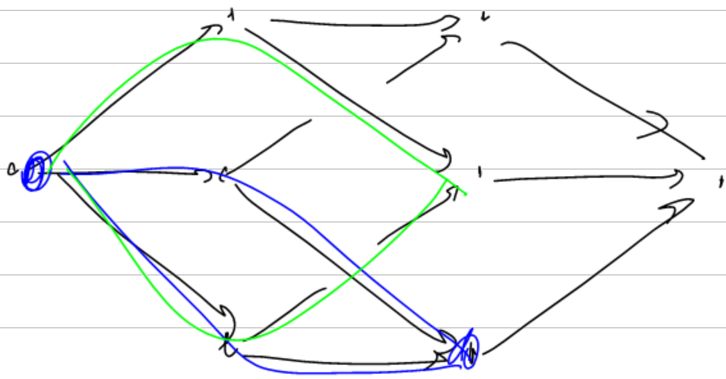
$$= (1-1)\cdots = 0$$

Q. If ${}_pa_p$, ${}_pb_q$, ${}_qc_p$, ${}_qd_q$ are matrices w/ undetermined coefficients, is $(a - bd^{-1}c)^{-1}$ always of degree $n = p+q$?

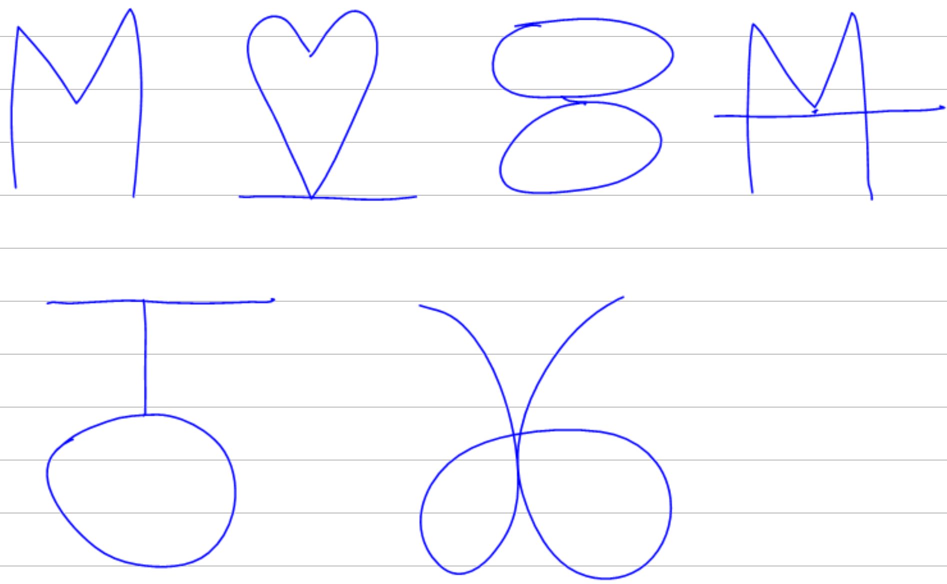
$(CA)^{-1} = C^{-1}A^{-1}$

$d^{-1} \sim \frac{q-1}{q}$ $\quad bd^{-1}c \sim \frac{q+1}{q}$

$a - bq^{-1}c \sim \frac{q+1}{q}$ $\quad (a-bq^{-1}c)^{-1} \sim \frac{(p-1)(q+1)+q}{p}$ wrong!



$m \quad \Delta$



$G \longrightarrow H$

$I \longrightarrow QG \longrightarrow QH$

Challenge: Do V_2 in less than L^8

V_1: Given $\alpha_i \in \mathbb{R}$, $s_i \in \{\pm 1\}$,

compute $$\sum_{i,j=1}^{L} s_i s_j \operatorname{sign}(i-j) \cdot \chi_{\alpha_i < \alpha_j}$$

in time $\ll L^2$.

Enough: $$\sum_{i,j=1}^{L} \chi_{i<j} \chi_{\alpha_i < \alpha_j}$$

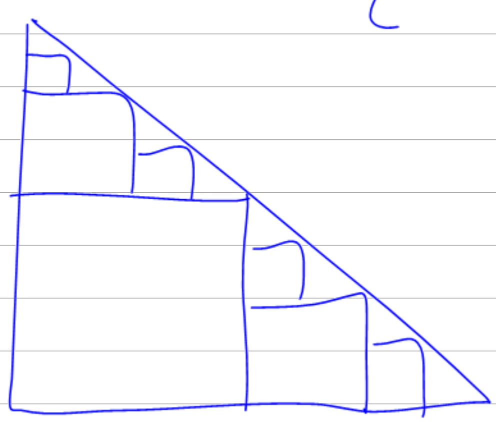
Easy!

V_2: Given α_i, β_i, compute

$$\sum_{i,j,k,l} \chi_{i<j} \chi_{k<l} \chi_{\alpha_i < \beta_j < \alpha_k < \beta_l}$$

similar & perhaps equiv to: Given $\sigma \in S_n$,

count $$\#\{i<j<k<l : \sigma_i < \sigma_j < \sigma_k < \sigma_l\}$$



Problem: Given $t_{i\alpha}\in\mathbb{R}$ all distinct, $1\le i\le k$, $1\le\alpha\le n$, quickly compute

$$\left|\left\{\begin{pmatrix}\alpha_1 & \dots & \alpha_k\\ \hat\beta_1 & \dots & \hat\beta_k\end{pmatrix} : t_{1\alpha_1}<t_{2\alpha_2}<\dots<t_{k\alpha_k}<t_{1\beta_1}<\dots<t_{k\beta_k}\right\}\right|$$

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\end{pmatrix}\qquad\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix}$$

$$\begin{pmatrix}f_1\\f_2\\1\end{pmatrix}$$

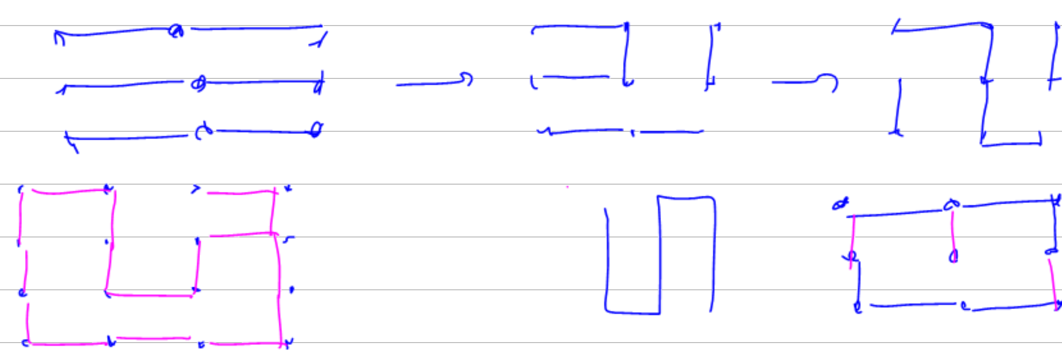
$$\begin{pmatrix}1&\epsilon_1&\epsilon_2\\\epsilon_4&1&\epsilon_3\\\epsilon_6&\epsilon_1&\epsilon_7\end{pmatrix}\begin{pmatrix}0\\0\\\sim\end{pmatrix}\begin{pmatrix}0\\1\\\sim\end{pmatrix}\begin{pmatrix}1\\0\\\sim\end{pmatrix}$$

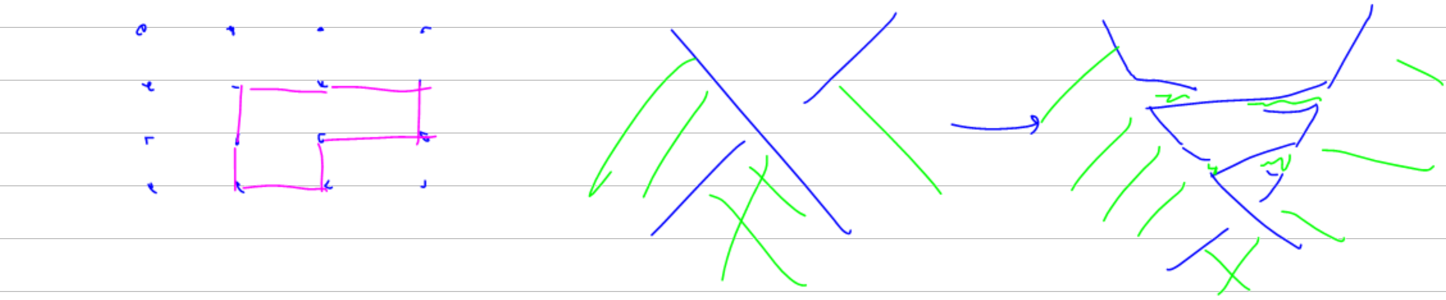
$$\begin{pmatrix}F_1(x_1,\dots,x_4)\\F_2(x_1,\dots,x_4)\\F_3(x_1\sim x_4)+x_3\\x_4\end{pmatrix}$$

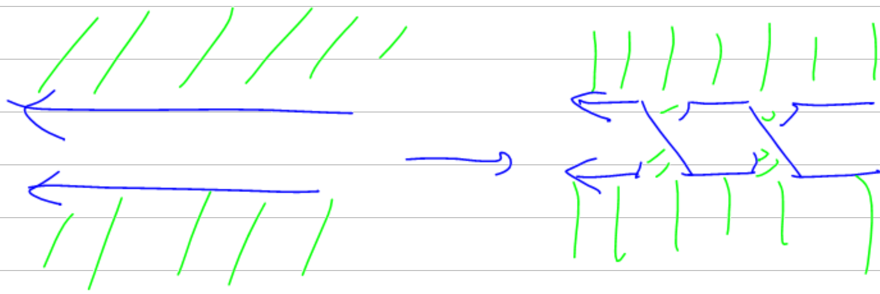
Alexander w/ n xings: $w^2\cdot n=n^2$ rops.

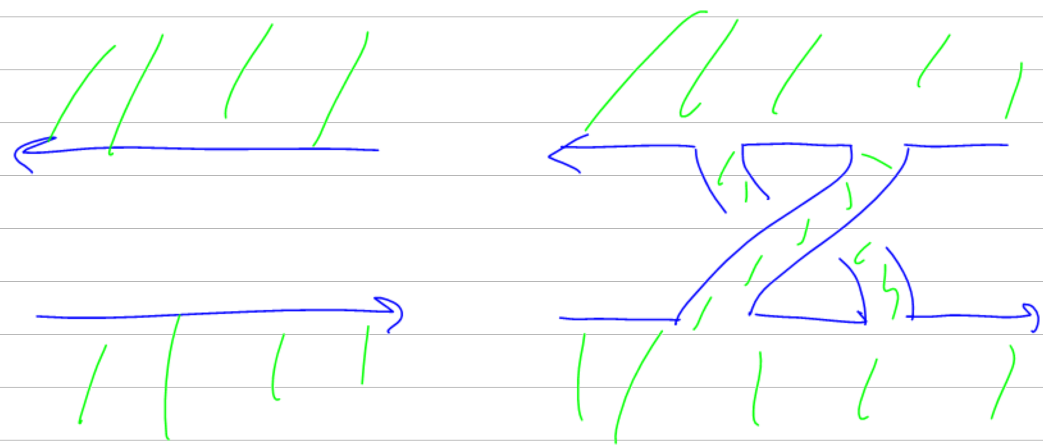
Alexander @ vol V: $\underbrace{V^3}_{\text{maybe}}$ rops. $\to (V^{\frac{4}{3}})^2$ rops.

$$(L^2)^2\cdot L^2$$









A counting problem:

given $\alpha_j, \beta_j \in [1..2d]$ $\quad j=1,\dots,d$

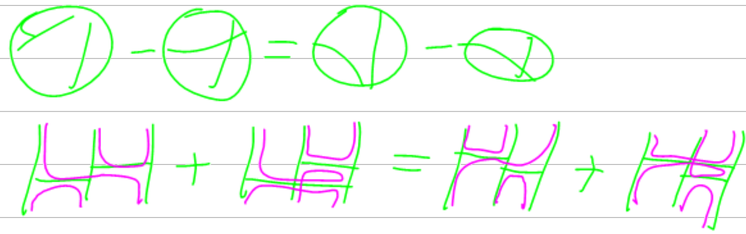
$2d$ "buckets" B_i $\quad [\,|B_i| \le L+1\,]$

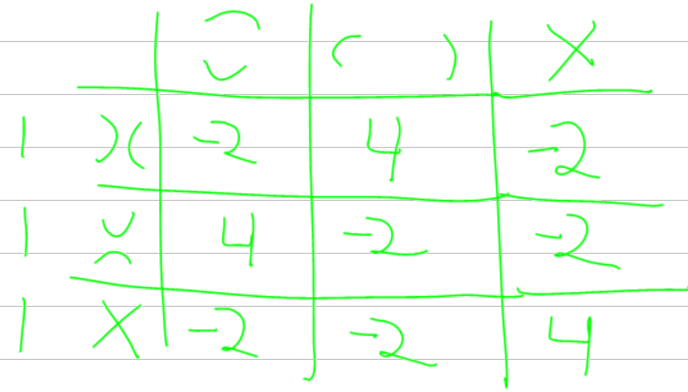
and functions $t:(B=\cup B_i)\longrightarrow\mathbb{Z}$ and

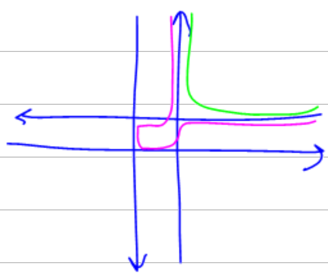
$z: B\longrightarrow[0..L]$, s.t. $z|_{B_i}$ is 1-1, compute

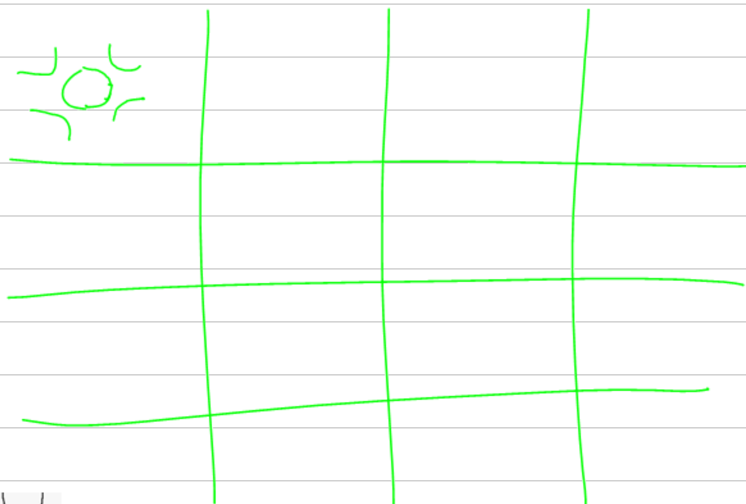
$$\left|\left\{ b=(b_i)_{i=1}^{2d}\in\prod B_i : \begin{array}{l} t(b_1)<t(b_2)<\dots<t(b_{2d}) \\ \forall j \quad z(b_{\alpha(j)})<z(b_{\beta(j)})\end{array}\right\}\right|$$

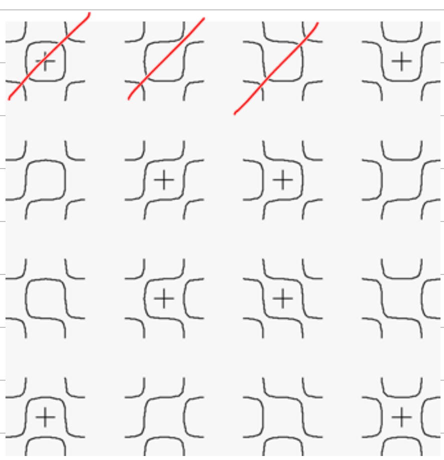
Claim (IBN): Can be done in time $\sim L^d$

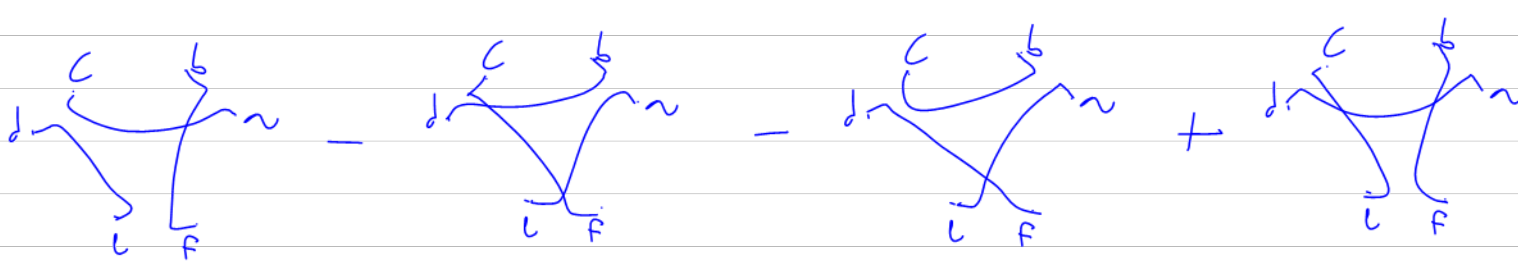












$$J(\bigcirc^k) = (q+q^{-1})^k$$

$$\left.\begin{aligned} J(\times) &= q\,J()() - q^2 J(\asymp) \\ J(\times) &= -q^{-2} J(\asymp) + q^{-1} J()() \end{aligned}\right\} W_2$$

$$\left. q^{-2} J(\times) - q^2 J(\times) = (q^{-1}-q)\, J(\uparrow\uparrow) \right\} W_1$$

$W_{1,2}$: $\bigcirc \longmapsto 2$

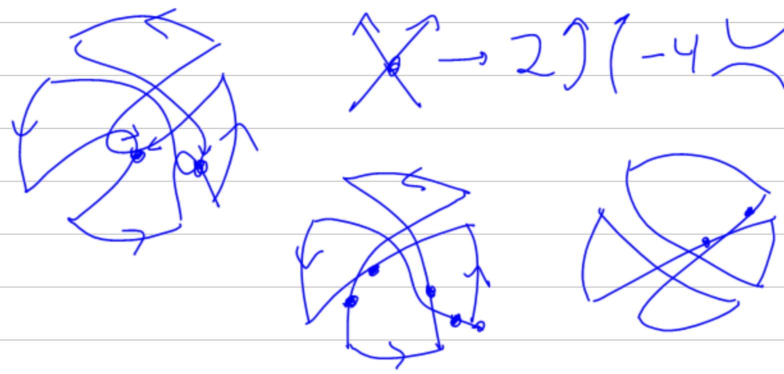
W_1: $\times \longmapsto 4\ \downarrow\ \uparrow - 2\ \asymp$ FI ✓

W_2: $\times \longmapsto -4\ \times + 2\ \asymp$ FI ✓

$$\otimes \xrightarrow{W_1} 16\,\bigcirc - 16\,\oslash + 4\,\infty = -24$$

$$\otimes \xrightarrow{W_2} 16\,(\cdot)_2 - 16\,(\cdot)_1 + 4\,\infty = 40$$

$$\times \longrightarrow 2\ )(- 4\ \asymp$$



$$z = (12345) = \overset{x = z^2}{(13524)}^3 = \overset{z^3 = y}{(14253)}^2$$

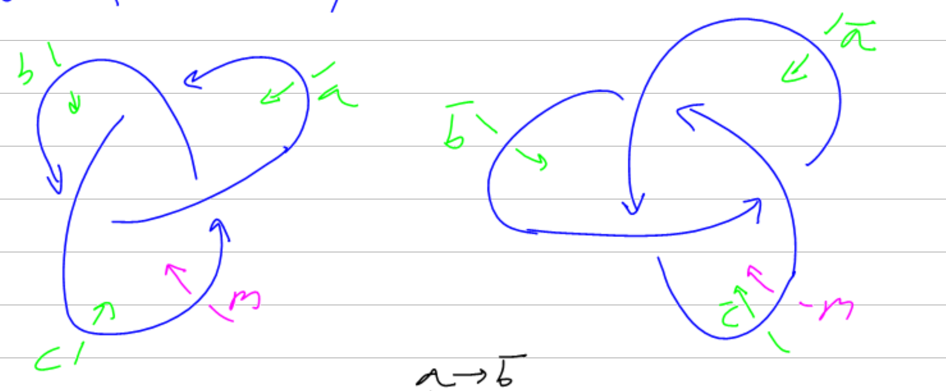
conj by $\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 3 & 4 & 5 & 1 & 2\end{pmatrix}$

$(13524) \qquad (142\qquad)$

$$yx^{-1} = (14253)(13524)^{-1} = (12345)$$

$$xy^{-1}x = (13524)(14253)^{-1}(13524) = (123\ldots$$

$$x^{-1}y = (123\ldots)$$



$a^b = c \quad b^c = a \quad c^a = b \quad \xrightarrow[c \to \bar c]{a \to \bar b,\ b \to \bar a} \quad \bar c = \bar b^{\bar a} \quad \bar a = \bar c^{\bar b} \quad \bar b = \bar a^{\bar c}$

$m = c \qquad m' = c \longleftarrow m = \bar c$

$l = acb \qquad l' = b^{-1}c^{-1}a^{-1} \qquad l = \bar a^{-1}\bar c^{-1}\bar b^{-1}$

$= l^{-1}$

$x = (135) \quad y = (12)(34)$ See Fox "Quick Tip" page 130.

$a^{-1} = x^{-1}y = (15432) \xrightarrow{\wedge b} (34125)$

$b^{-1} = xy^{-1}x = (14523) \xrightarrow{\wedge c} (43215)$

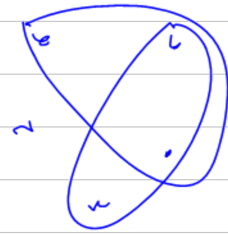
$c^{-1} = yx^{-1} = (12534) \xrightarrow{\wedge a} (23145)$

$l = acb = (12345)(43521)(32541) = (13245)$

4513

$m = (14352)$

$$\underset{y}{\underset{\parallel}{(1234)}}^2 = (13)(24) = \Big(\underset{x}{\underset{\parallel}{(13)(24)}}\Big)^3$$



$$xy^{-1} = (13)(24)\,(1234)^{-1} = (1234)$$

$$y^{-1}x = (1234)^{-1}(13)(24) = (1234)$$

$(12)(34)$ $\quad$ $(13)(24)$ $\quad$ $(14)(23)$

$(123)(45)$

$$\left(\partial_x e^{\xi\partial_x}\right)\frac{1}{x}\Big|_{x=0} = \partial_x \frac{1}{x+\xi}\Big|_{x=0}$$

$$= -\frac{1}{(x+\xi)^2}\Big|_{x=0} = -\frac{1}{\xi^2}$$

$$\partial\left(Pe^Q\right) = \left(P'+PQ'\right)e^Q \qquad P'+PQ' = R$$

$$\partial^{-1}\left(Re^Q\right) = \left(\qquad\qquad\right)e^Q \qquad P_1 = \frac{R}{Q'}$$

$$P_1' + P_1 Q' - R = P_1' = \frac{R'Q'-RQ''}{(Q')^2}$$

$\varphi_a(x) := x \wedge a$

$\varphi_{a \wedge b} \overset{?}{=} \varphi_b^{-1} \varphi_a \varphi_b$

$\varphi_b \varphi_{a \wedge b} \overset{?}{=} \varphi_a \varphi_b$

$(x \wedge (a \wedge b)) \wedge b \overset{?}{=} (x \wedge b) \wedge a$

$(x \wedge b) \wedge (a \wedge b) = (x \wedge a) \wedge b$

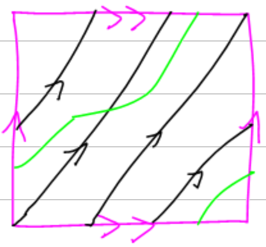
$a \wedge b = -a-b$

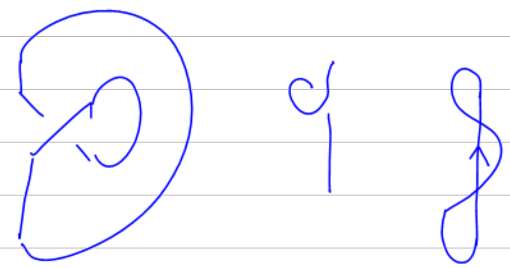
$(a \wedge b) \wedge c = (-b-a) \wedge c = a+b-c$

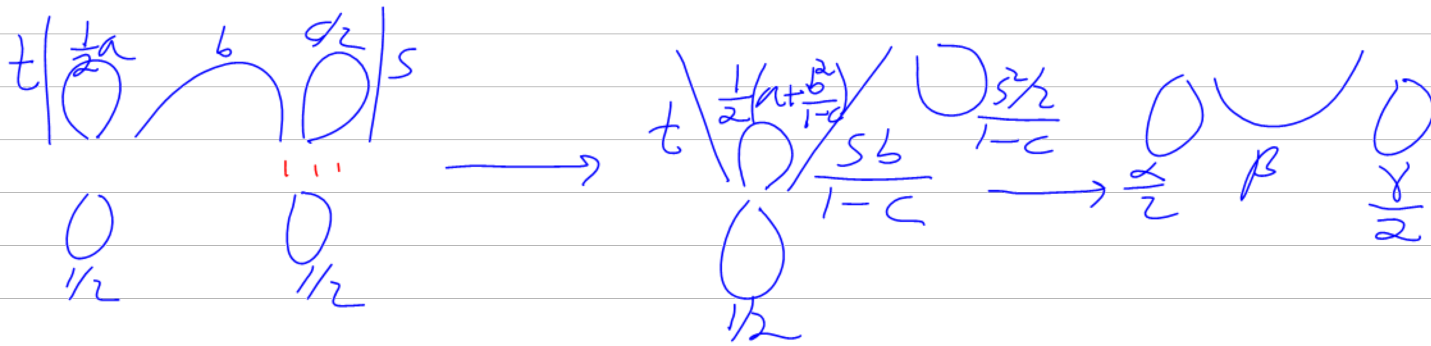
$(a \wedge c) \wedge (b \wedge c) = (c-a) \wedge (c-b) = a+b+2c$

$\varphi_a = \varphi_b$

$a = a \wedge b$







$$\alpha = t^2 \frac{1}{1-\left(a+\frac{b^2}{1-c}\right)} = t^2 \frac{1-c}{(1-a)(1-c)-b^2} \qquad \beta = t\frac{sb}{1-c}\cdot\frac{1}{1-\left(a+\frac{b^2}{1-c}\right)} = \frac{tsb}{(1-a)(1-c)-b^2}$$

$$\gamma = \frac{s^2b^2}{(1-c)^2}\frac{1}{1-\left(a+\frac{b^2}{1-c}\right)} + \frac{s^2}{1-c} = \frac{s^2b^2+s^2[(1-a)(1-c)-b^2]}{(1-c)[(1-a)(1-c)-b^2]} = \frac{s^2(1-a)\cancel{(1-c)}}{\cancel{(1-c)}[(1-a)(1-c)-b^2]}$$