



Problem 1 (40 points). Let K be a knot in \mathbb{R}^3 presented by a planar diagram D . With a massive use of van Kampen's theorem, show that the fundamental group of the complement of K has a presentation (the “Wirtinger presentation”, as discussed in class) with one generator for each edge of D and two relations for each crossing of D , as indicated in the figure below.

A generator for each edge

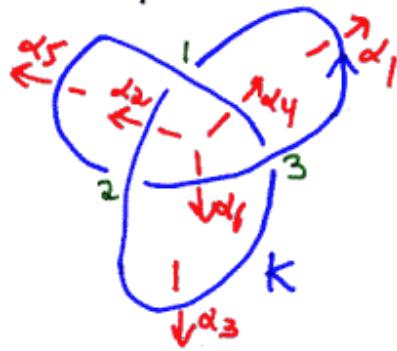
$$\alpha \rightarrow \langle \dots \alpha \dots \rangle$$

Two relations for each crossing

$$\alpha = \gamma$$

$$\alpha \beta = \delta \gamma$$

Example:



$$\pi_1(K^c) = \langle \alpha_1, \dots, \alpha_6 \rangle$$

$$\left. \begin{array}{l} \alpha_4 = \alpha_5 \\ \alpha_4 \alpha_1 = \alpha_2 \alpha_5 \end{array} \right\} 1$$

$$\left. \begin{array}{l} \alpha_2 = \alpha_3 \\ \alpha_2 \alpha_5 = \alpha_6 \alpha_3 \end{array} \right\} 2$$

$$\left. \begin{array}{l} \alpha_6 = \alpha_1 \\ \alpha_6 \alpha_3 = \alpha_4 \alpha_1 \end{array} \right\} 3$$

renaming $\alpha_1 = \alpha$; $\alpha_5 = \beta$; $\alpha_3 = \gamma$ we get

$$\pi_1(K^c) = \langle \alpha, \beta, \gamma \rangle / \gamma^\beta = \alpha, \beta = \alpha^\gamma, \beta^\alpha = \gamma$$

Problem 2 (20 points). The trefoil knot above, whose fundamental group is $G_1 = \langle \alpha, \beta, \gamma: \alpha = \gamma^\beta, \beta = \alpha^\gamma, \gamma = \beta^\alpha \rangle$ is in fact the torus knot $T_{3/2}$, whose fundamental group, as computed in class, is $G_2 = \langle \lambda, \mu: \lambda^2 = \mu^3 \rangle$. Prevent the collapse of mathematics by showing that these two groups are isomorphic.