# Homework Assignment 2

**Problem 1.** A topological space $X$ is called "locally path connected" if every point in it has arbitrarily small neighborhoods that are path connected. Namely, if $x \in X$ and if $U$ is a neighborhood of $x$, then there is a path connected open set $V$ such that $x \in V \subset U$.

*On the subject of liftings:* Prove that if $X$ is path connected, locally path connected, and simply connected, and if $p\colon (E, e_0) \to (B, b_0)$ be a covering map, then every $f\colon (X, x_0) \to (B, b_0)$ has a unique lift to a map $\tilde{f}\colon (X, x_0) \to (E, e_0)$ such that $\tilde{f} /\!\!/ p = f$.

*Hint.* Lift paths, lift endpoints of paths, worry about well-definedness, worry about continuity.

*On the right.* A space that is path-connected but not locally path-connected.

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Graphics[{Line[{{1, 0}, {0, 0}, {0, 1}}],
  Table[Line[{{1/n, 0}, {1/n, 1}}], {n, 1000}]}]
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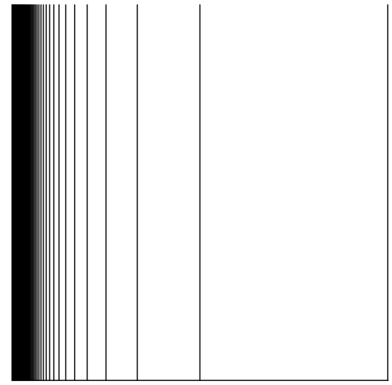

**Problem 2.** With the obvious assumptions and definitions, prove that $\pi_1((X, x_0) \times (Y, y_0)) \cong \pi_1(X, x_0) \times \pi_1(Y, y_0)$.

**Problem 3.** Prove that if $\gamma\colon S^1 \to S^1$ is even (meaning, $\forall z \in S^1\ \gamma(-z) = \gamma(z)$) then $[\gamma]$, an integer, is even. Likewise, prove that if $\gamma\colon S^1 \to S^1$ is odd (meaning, $\forall z \in S^1\ \gamma(-z) = -\gamma(z)$) then $[\gamma]$ is odd.

*Hint.* Re-interpret $\gamma$ as a map with domain $[0, 1]$ with some symmetry property. In both cases, $\gamma$ is determined by its values on $[0, \frac{1}{2}]$, and if the lift $\bar{\gamma}$ of $\gamma|_{[0,\frac{1}{2}]}$ is known then the full lift $\tilde{\gamma}$ of $\gamma|_{[0,1]}$ is determined by it. What can you say about $\bar{\gamma}(\frac{1}{2})$? What does it say about $\tilde{\gamma}(1)$?

**Problem 4.** Let $M$ denote the Möbius band.

1. Show that $\pi_1(M) \cong \mathbb{Z}$.
2. Is there a retraction from $M$ to its boundary?