Homework Assignment 1

Problem 1. Write explicit formulas for the homotopy in https://drorbn.net/bbs/show?shot=26-1301-260105-110227.jpg between e and $\gamma\bar{\gamma}$ (and if that picture is wrong, fix it in your mind first). Your solution should take up 3 lines and must be of the form:

$$h(t,s) = \begin{cases} \text{formula 1} & \text{condition 1} \\ \text{formula 2} & \text{condition 2} \\ \text{formula 3} & \text{condition 3} \end{cases}$$

Problem 2. Prove the theorem which is implicit in the definition at https://drorbn.net/bbs/show?shot=26-1301-260106-163930.jpg and https://drorbn.net/bbs/show?shot=26-1301-260106-163954.jpg. Namely, prove that if X is path-connected then the following are equivalent:

1. $\pi_1(X, x_0) = 0$ for some/any $x_0 \in X$.
2. If γ_0 and γ_1 are not-necessarily-closed paths that share their endpoints, namely $\gamma_0(0) = \gamma_1(0) = x$ and $\gamma_0(1) = \gamma_1(1) = y$, then they are homotopic via a homotopy that does not move these endpoints.
3. Any two maps $S^1 \to X$ are homotopic.

Problem 3. If X is path-connected, prove that the set of homotopy classes of maps $S^1 \to X$ can be put in a bijection with the set of conjugacy classes in the fundamental group of X.