Construction: For $v \in V$, $\sigma \mapsto C_v\sigma$ by

$$\sigma = [u_0 \dots u_n] \mapsto [v, u_0 \dots u_n]$$

$$\Rightarrow \partial C_v\sigma = \sigma - C_v\partial\sigma \qquad \Big| \; b\sigma := \frac{1}{n+1}\sum u_i$$

"The barycentre"

$$S\sigma := \begin{cases} \sigma & n=0 \\ C_{b\sigma}(S\partial\sigma) & n>0 \end{cases} \qquad T\sigma := \begin{cases} \cancel{0} - C_{b\sigma}\sigma & n=0 \\ C_{b\sigma}(S\sigma - \sigma - T\partial\sigma) & n>0 \end{cases}$$

$\partial S = S\partial$: $\partial S\sigma = \partial(C_{b\sigma}(S\partial\sigma)) = S\partial\sigma - C_{b\sigma}\partial S\partial\sigma = S\partial\sigma$

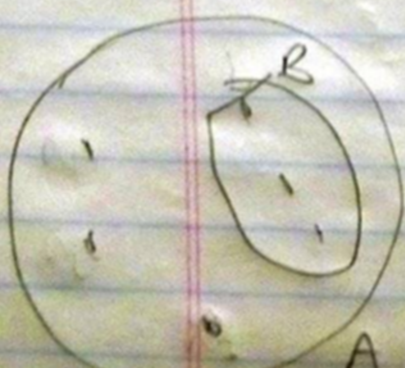
$S - I = \partial T + T\partial$: $\partial T\sigma = \partial C_{b\sigma}(S\sigma - \sigma - T\partial\sigma) =$

$$= S\sigma - \sigma - T\partial\sigma - C_{b\sigma}\big(\underbrace{\cancel{\partial S\sigma - \partial\sigma - \partial T\partial\sigma}}_{\text{induction}}\big)$$

<u>Claim</u> If τ appears in $S\sigma$, then $\operatorname{diam}\tau < \frac{n}{n+1}\operatorname{diam}\sigma$

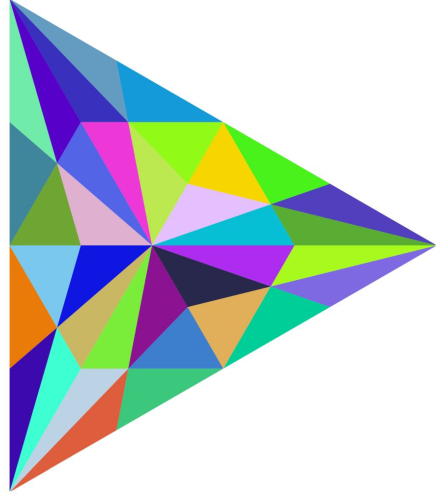
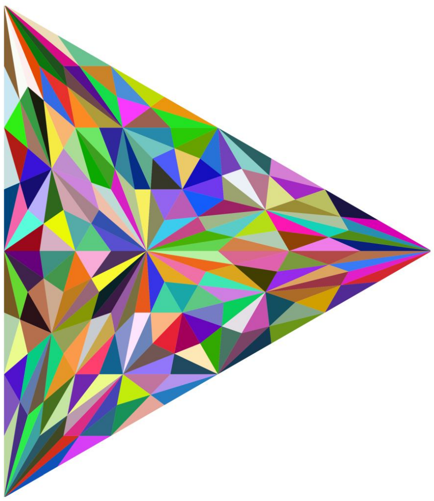
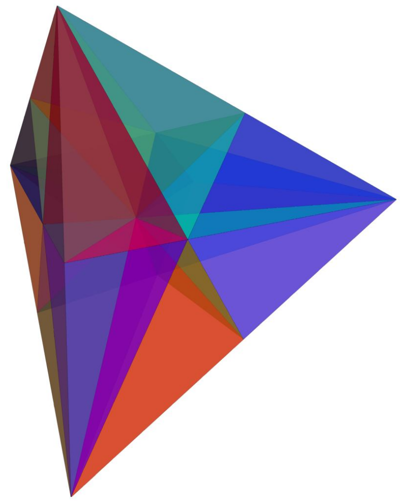
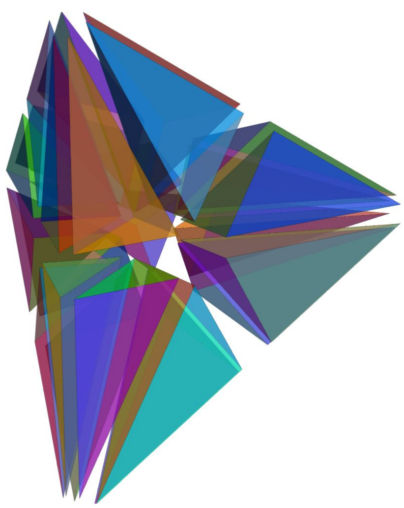
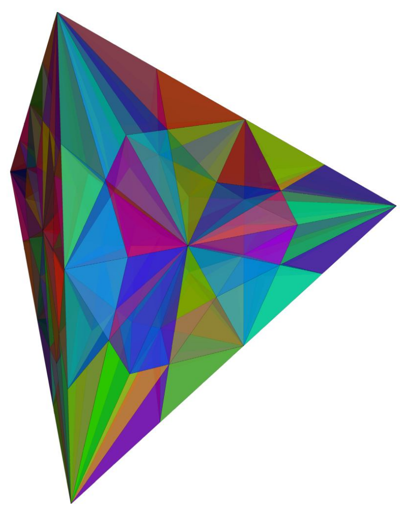
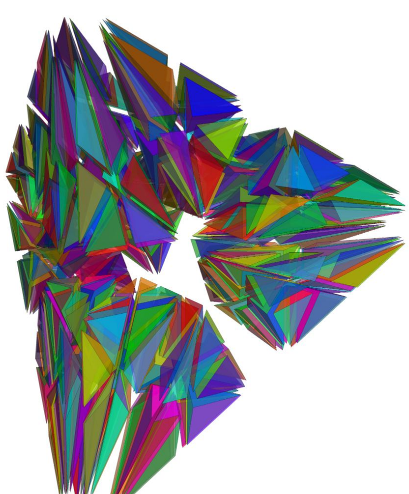
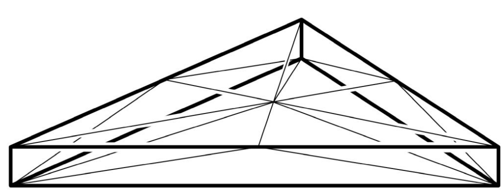
<u>Lemma</u> If $B \subset A$ are finite non-empty sequences of vectors,

$$|bB - bA| \le \frac{|A|-1}{|A|}\max_{v,w\in A}|v-w|$$



<u>Pf</u> 1. replace B by $|B|$ times bB;
lhs stays, rhs shrinks.
2. Replace $A-B$ by $(|A|-|B|)$ times $b(A-B)$

$\square$

Hatcher's "Algebraic Topology", Page 122