MAT347 TUTORIAL

(1) **T/F**: If $(R_1, +_1, \times_1)$ and $(R_2, +_2, \times_2)$ are rings and $f\colon (R_1, +_1) \to (R_2, +_2)$ is group homomorphism then f is a ring homomorphism.
(2) **T/F**: If $(R_1, +_1, \times_1)$ and $(R_2, +_2, \times_2)$ are rngs and $f\colon (R_1, +_1) \to (R_2, +_2)$ is group homomorphism then f is a rng homomorphism.
(3) **T/F**: If $(R_1, +_1, \times_1)$ and $(R_2, +_2, \times_2)$ are rings and $f\colon (R_1, +_1) \to (R_2, +_2)$ is group isomorphism then f is a ring isomorphism.
(4) **T/F**: If $(R_1, +_1, \times_1)$ and $(R_2, +_2, \times_2)$ are rngs and $f\colon (R_1, +_1) \to (R_2, +_2)$ is group isomorphism then f is a rng isomorphism.
(5) **T/F** If $f\colon (\mathbb{R}, +) \to (\mathbb{R}, +)$ is a group homomorphism then f is a ring homomorphism.
(6) **T/F** If $I \subset R$ is an ideal then $(I, +) \triangleleft (R, +)$.
(7) **T/F** If $\mathbb{F}$ is a field and $I \subset \mathbb{F}$ is an ideal then $I = 0$.
(8) **T/F** If $\mathbb{F}$ is a field, $R = M_2(\mathbb{F})$ (2×2 matrices with entries in $\mathbb{F}$) and $I \subset R$ is an ideal then $I = 0$.
(9) **T/F** There exists a ring homomorphism $M_2(\mathbb{R}) \to \mathbb{R}$.
(10) **T/F** If $f\colon R_1 \to R_2$ is a ring homomorphism and $I \subset R_1$ is an ideal then $f(I)$ is an ideal.

If R is a ring the **characteristic** of R, denoted $\mathrm{Char}(R)$, is the smallest integer n such that $\sum_{i=1}^{n} 1 = 0$. If no such integer exists then we define $\mathrm{Char}(R) = 0$.

(11) **T/F** If R is an integral domain then $\mathrm{Char}(R) = 0$ or $\mathrm{Char}(R) = |R|$.
(12) **T/F** If R is a field then $\mathrm{Char}(R)$ is prime.
(13) **T/F** If R is an integral domain and $\mathrm{Char}(R)$ is prime then $|R| = \mathrm{Char}(R)$.
(14) **T/F** If R is an integral domain and $\mathrm{Char}(R)$ is prime then R is a field.
(15) **T/F** If R is an integral domain and $\mathrm{Char}(R)$ is prime then R is a field.
(16) **T/F** If R is a finite field then $\mathrm{Char}(R)$ is prime.
(17) **T/F** If $\mathrm{Char}(R)$ is prime then R is a finite field.