

## MAT347 TUTORIAL 6

- (1) Suppose  $G$  is a finite group and  $N$  is a normal subgroup such that  $|N|$  and  $|G/N|$  are coprime. Prove that  $N$  is the only subgroup with order  $|N|$ .
- (2) Let  $G$  be a finite group and let  $f \in \text{Aut}(G)$  have order 2 and no non-identity fixed points. Prove that  $G$  is abelian.
- (3) If  $G$  is a finite group and  $H$  is a proper subgroup prove that  $\bigcup_{g \in G} gHg^{-1} \neq G$ .
- (4) If  $G$  is a finite group acting transitively on a set  $X$  such that  $|X| > 1$  prove there exists  $g \in G$  with no fixed points.
- (5) Classify all groups of order  $pq$  where  $p < q$  are prime.