

MAT347 TUTORIAL 6

- (1) Suppose G is a finite group and N is a normal subgroup such that $|N|$ and $|G/N|$ are coprime. Prove that N is the only subgroup with order $|N|$.
- (2) Let G be a finite group and let $f \in \text{Aut}(G)$ have order 2 and no non-identity fixed points. Prove that G is abelian.
- (3) If G is a finite group and H is a proper subgroup prove that $\bigcup_{g \in G} gHg^{-1} \neq G$.
- (4) If G is a finite group acting transitively on a set X such that $|X| > 1$ prove there exists $g \in G$ with no fixed points.
- (5) Classify all groups of order pq where $p < q$ are prime.