MAT347 TUTORIAL 2

Define the *center* of a group $\text{Cent}(G) = \{g \in G : [g, h] = e \text{ for all } h \in G\}$.

(1) Prove that $\text{Cent}(G)$ is abelian.
(2) Prove that $G = \text{Cent}(G)$ if and only if G is abelian.
(3) Show that if $G/\text{Cent}(G)$ is cyclic then G is abelian.
(4) Recall the map $C\colon G \to \text{Aut}(G)$, $C_h(g) = h^{-1}gh$. The image of C is a normal subgroup denoted $\text{Inn}(G)$. Show that $G \cong \text{Inn}(G)$ if and only if $\text{Cent}(G) = \{e\}$. What is $\ker(C)$?
(5) Give an example of a group G such that $\text{Aut}(G) \ncong G$
(6) Give an example of a group G such that $\text{Aut}(G) = \text{Inn}(G)$.
(7) Find examples of groups G, H such that $|G| > |H|$ but $|\text{Aut}(G)| < |\text{Aut}(H)|$.
(8) Find examples of cyclic groups G, H such that $|G| > |H|$ but $|\text{Aut}(G)| < |\text{Aut}(H)|$.