MAT347 TUTORIAL

(1) What is the rank of $\mathbb{Z}/m\mathbb{Z}$ as a $\mathbb{Z}/m\mathbb{Z}$-module?
(2) What is the rank of $\mathbb{Z}/m\mathbb{Z}$ as a $\mathbb{Z}$-module?
(3) What is the rank of $(2, x) \subset \mathbb{Z}[x]$ as a $\mathbb{Z}$-module?
(4) What is the rank of $(2, x) \subset \mathbb{Z}[x]$ as a $\mathbb{Z}[x]$-module?
(5) Prove that $\mathbb{Q} \otimes_{\mathbb{Z}} \mathbb{Q} \cong \mathbb{Q}$.
(6) Find $\mathbb{Q} \otimes_{\mathbb{Z}} (\mathbb{Q}/\mathbb{Z})$
(7) What is $\mathbb{Z}/m\mathbb{Z} \otimes_{\mathbb{Z}} \mathbb{Z}/n\mathbb{Z}$?
(8) Let $M = \mathbb{R}^2 \otimes_{\mathbb{R}} \mathbb{R}^2$ and let $e_1 = (1, 0)$ and $e_2 = (0, 1)$ in $\mathbb{R}^2$. Prove that $e_1 \otimes e_2 + e_2 \otimes e_1$ is not a simple tensor (i.e. it isn't equal to $v \otimes w$ for $v, w \in \mathbb{R}^2$).
(9) Let $\mathbb{F}$ be a field, V a finite dimensional vector space over $\mathbb{F}$, and $S \colon V \to V$ a linear transformation. We turn V into a $\mathbb{F}[x]$-module V_S in the following way. For $p \in \mathbb{F}[x]$ write $p = \sum_{i=0}^{n} a_i x^i$ and define $p_S \colon V \to V$ by $p_S(v) = \sum_{i=0}^{n} a_i S^i(v)$. With this module structure prove that $T \colon V_S \to V_S$ is a $\mathbb{F}[x]$-module homomorphism if and only if T is a linear transformation that commutes with S.