MAT347 TUTORIAL

(1) True or false: $\mathbb{Z}[x]/(3, x^2 - 1) \cong (\mathbb{Z}/3\mathbb{Z})/(x^2 - 1)$.
(2) Prove that $(2, x^2 - 1)$ is not prime in $\mathbb{Z}[x]$.
(3) Find the GCD of $x^3 - 8x^2 + 17x - 10$ and $x^3 - 5x^2 + 7x - 3$ in $\mathbb{Z}[x]$.
(4) Show that the map $N \colon \mathbb{Z}[\sqrt{6}] \to \mathbb{Z}$ defined by $N(a + b\sqrt{6}) = a^2 - 6b^2$ satisfies $N(ab) = N(a)N(b)$.
(5) If $z \in \mathbb{Z}[\sqrt{6}]$ is a unit what is $N(z)$?
(6) Find $N^{-1}(2)$, $N^{-1}(3)$, and $N^{-1}(6)$.
(7) Prove that 2 and 3 are irreducible in $\mathbb{Z}[\sqrt{6}]$.
(8) We can write $6 = (\sqrt{6})^2 = 2 \times 3$. Why is this not enough to conclude $\mathbb{Z}[\sqrt{6}]$ is not a UFD? (In fact, $\mathbb{Z}[\sqrt{6}]$ *is* a UFD)