This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

Homework Assignment 12   ⑦

Solve and submit your solutions of the following problems. Note also that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Due date
Friday, January 30, 2026 11:59 pm (Eastern Standard Time)

Late penalty
5% deducted per hour

Q1 (10 points)

Find the tensor product of the $\mathbb{C}[t]$ modules $\mathbb{C}[t, t^{-1}]$ ("Laurent polynomials in t") and $\mathbb{C}$ (here t acts on $\mathbb{C}$ as 0).

Q2 (10 points)

Show that over any ring R, the tensor product of the R-module $R[x]$ with itself is the R-module $R[x, y]$.

Q3 (10 points)

Let V and W be finite dimensional vector spaces over a field F, and let $\mathrm{Hom}(V, W)$ denote the vector space of all linear transformations $V \to W$.

- Show that $\mathrm{Hom}(V, W)$ is isomorphic to $V^* \otimes W$ via a *natural* isomorphism. (The phrase *natural* has a formal definition within category theory, but stating it will take us too far away. So let's say that I just mean, "an isomorphism that does not depend on making any choices, such as a basis for V or for W).

- The assumptions in this question can be relaxed a bit, and one V and W can be allowed to be infinite dimensional. Which of them is it?

Q4 (10 points)

Let V be a finite dimensional vector space over a field F. There is an obvious bilinear map $e : V^* \times V \to F$, defined by $(\varphi, v) \mapsto \varphi(v)$. The functional definition of tensor products implies that there a unique linear map $t : V^* \otimes V \to F$ such that $t \circ \iota = e$, where ι is the standard map $V^* \times V \to V^* \otimes V$. In the light of the previous question, t can be interpreted as a linear map $t : \mathrm{Hom}(V, V) \to F$. Can you identify t as one of the well-known standard linear algebra operations? Which one? Why?

Ready to submit?

→   Please ensure all pages are in order and rotated correctly before you submit

→   You will not be able to resubmit your work after the due date has passed.

$\rightleftarrows$ Please wait...