This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

Homework Assignment 7

Solve and submit your solutions of the following problems. Note also that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Due date

Friday, November 14, 2025 11:59 pm (Eastern Standard Time)

Late penalty

5% deducted per hour

Q1 (10 points)

Let N and H be groups, and let $\phi : H \to \mathrm{Aut}(N)$ and $\psi \in \mathrm{Aut}(H)$ be given, and let $\phi' := \phi \circ \psi$. Show that $N \rtimes_{\phi'} H$ is isomorphic to $N \rtimes_{\phi} H$.

Note. "$\psi \in \mathrm{Aut}(H)$" is *correct*.

Q2 (10 points)

Let $N = \langle x \rangle / \langle x^7 \rangle$ and let $H = \langle y \rangle / \langle y^3 \rangle$, and let $\phi_1 : H \to \mathrm{Aut}(N)$ be given by $\phi_{1y}(x) = x^2$ and $\phi_2 : H \to \mathrm{Aut}(N)$ be given by $\phi_{2y}(x) = x^4$. Show that the groups $N \rtimes_{\phi_i} H$ for $i = 1, 2$ are isomorphic.

Q3 (10 points)

Let N, A, and B be groups, and let $H := A \times B$. Suppose $\psi : A \to \mathrm{Aut}(N)$ is given, and let $\phi := \psi \circ \pi_A$ where $\pi_A : A \times B \to A$ is the projection on the first factor. Show that $N \rtimes_\phi H$ is isomorphic to $(N \rtimes_\psi A) \times B$.

Note. An earlier version of this question had "$\psi : A \to \mathrm{Aut}(H)$". The correct version is as it is now, "$\psi : A \to \mathrm{Aut}(N)$".

Q4 (10 points)

Show that up to isomorphism and other then the direct product, there is only one group of the form $\mathbb{Z}/3 \rtimes (\mathbb{Z}/2 \times \mathbb{Z}/2)$.