Homework Assignment 8

Problem 1.

1. Formulate and prove a naturality property for the Mayer-Vietoris sequence. Your property must be at least strong enough to answer part 2 of this question.

2. Use part 1 of this question to prove that if $f\colon S^n \to S^n$ then $\deg(f) = \deg(\Sigma f)$ where Σ is the suspension functor, mentioned previously both in class and in HW7.

Problem 2. Suppose n is even.

1. Show that for any continuous map $f\colon S^n \to S^n$ there is a point x such that $f(x) = \pm x$.

2. Show that any continuous map $f\colon \mathbb{R}P^n \to \mathbb{R}P^n$ has a fixed point.

Problem 3.

1. Compute the homology over $\mathbb{Z}$ of a the space X obtained from the 2D disk D^2 by identifying each of its boundary points with the point you get from it by applying a 1/3 rotation counterclockwise.

2. Same question, but over $\mathbb{Z}/3$.

Problem 4. The statement "all reasonable everyday spaces are at least homotopy equivalent to CW complexes" sounds completely reasonable. At least until you hit the first example where it's hard.

Show that the complement X of the trefoil knot  in $\mathbb{R}^3$ is homotopy equivalent to a 3-dimensional CW complex.