**Problem 1.** A topological space X is called "locally path connected" if every point in it has arbitrarily small neighborhoods that are path connected. Namely, if $x \in X$ and if U is a neighborhood of x, then there is a path connected open set V such that $x \in V \subset U$.

On the subject of liftings: Prove that if X is path connected, locally path connected, and simply connected, and if $p \colon (E, e_0) \to (B, b_0)$ be a covering map, then every $f \colon (X, x_0) \to (B, b_0)$ has a unique lift to a map $\tilde{f} \colon (X, x_0) \to (E, e_0)$ such that $\tilde{f} /\!\!/ p = f$.

Hint. Lift paths, lift endpoints of paths, worry about well-definedness, worry about continuity.

On the right. A space that is path-connected but not locally path-connected.

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Graphics[{Line[{{1, 0}, {0, 0}, {0, 1}}],
  Table[Line[{{1 / n, 0}, {1 / n, 1}}], {n, 1000}]}]
```
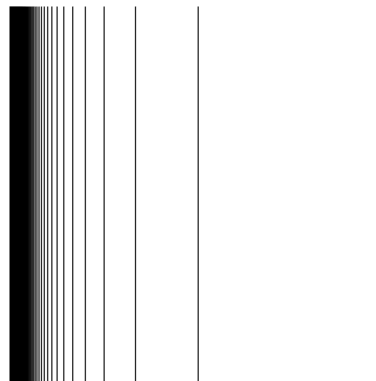

Problem 2. With the obvious assumptions and definitions, prove that $\pi_1((X, x_0) \times (Y, y_0)) \cong \pi_1(X, x_0) \times \pi_1(Y, y_0)$.

Problem 3. Let M denote the Möbius band.

1. Show that $\pi_1(M) \cong \mathbb{Z}$.

2. Is there a retraction from M to its boundary?