Do not turn this page over until instructed.

Final Exam

University of Toronto, April 9, 2025

Respond to all 4 tasks on the other side of this page.
The tasks are of equal weight.
You have three hours to write this test.

Notes

- No outside material allowed other than stationery, minimal hydration and snacks, and stuffed animals.

- Do not write on this form.

- Three of the four tasks on this form are not-fully-specified "write an essay" tasks. This is a graduate class, and one of the greatest challenges you will be facing later on will be to free-style write rather than demonstrate to a TA that you've memorized some material. So the essay tasks do not specify precisely what you need to do – it is up to you to decide what is important and what isn't, which figures to draw and when are they superior to formulas, which formulas to write and when are they superior to figures, etc. Think that you really are writing a paper and you really are trying to transmit information that will be read and understood.

- **Neatness counts! Language counts!** You may want to write "draft essays" and only then write the final versions of your solutions. Messy essays cannot be read.

- **In Red.** Comments added after the test took place.

Good Luck!

Task 1. Let X be the "Olympic Rings" covering of the figure 8 space, 8^a_b, whose basepoint is taken to be at the quadrivalent vertex in its centre and whose fundamental group is the free group on two letter a and b: $G := \pi_1(8^a_b) = F(a,b)$.
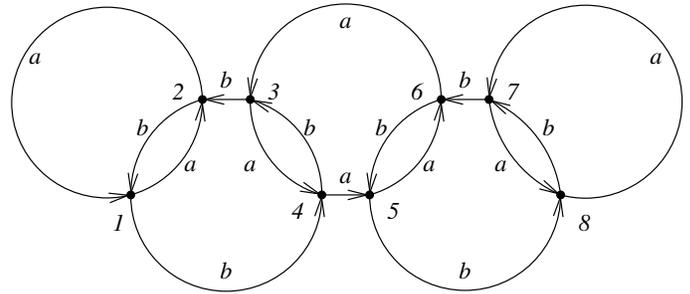
A. In your examination notebook (not here), describe the right G-set S corresponding to $8^a_b \, X$: S is a set with __ elements, and a and b act on it as the permutations ____ and ____.

B. Taking the basepoint x_1 of X to be the point marked as "1" on the right, write a set of generators for the image H of $\pi_1(X, x_1)$ within $G := \pi_1(8^a_b)$.

C. Is H a normal subgroup of G?

Task 2. Write an essay on "the prism construction". What is it? How is it used? Who cares?

Task 3. Write an essay on "the transfer sequence for the $\mathbb{Z}/2$ homology of a double cover and the homology of projective spaces". What is it? How is it used? Who cares?

Task 4. Write an essay on "the degrees of maps $S^n \to S^n$". What are they? What are their properties? Who cares?

Good Luck!

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