

Questions

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Assignment description

Edit Preview

Solve and submit your solutions of the following problems. Note that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Attach files i Formatting tips

Q1 Image/PDF question

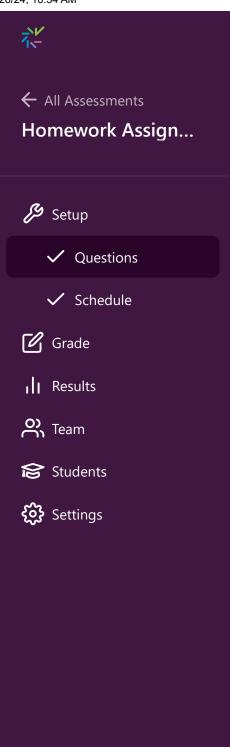
0 points

Read sections 51-55 in Munkres' textbook (Topology, 2nd edition). Remember that reading math isn't like reading a novel! If you read a novel and miss a few details most likely you'll still understand the novel. But if you miss a few details in a math text, often you'll miss everything that follows. So reading math takes reading and rereading and rerereading and a lot of thought about what you've read.

Q2 Image/PDF question

10 points

Show that the two definitions given in class for a covering $p:E \to B$ are equivalent:



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Definition 1. There is an open cover \mathcal{U} of B such that for every $U \in \mathcal{U}$ there is a discrete set D and a homeomorphism $\phi : U \times D \to p^{-1}(U)$ such that $p \circ \phi = \pi_U$, where $\pi_U : U \times D \to U$ is the projection on the first component.

Definition 2. There is an open cover \mathcal{U} of B such that for every $U \in \mathcal{U}$, its inverse image $p^{-1}(U)$ is a union of disjoint open sets U_{β} in E such that for each β the restriction of p to U_{β} is a homeomorphism of U_{β} with U.



Question

Edit **Preview**

A space X is called "locally path connected" if for every $x \in X$ and every open set $U \subset X$ with $x \in U$, there is a path-connected open set V such that $x \in V \subset U$.

Show that if $p: (E, e_0) \to (B, b_0)$ is a covering, if (X, x_0) is path connected, locally path connected, and simply connected and if $\psi: (X, x_0) \to (B, b_0)$ is given, then there is a unique $\tilde{\psi}: (X, x_0) \to (E, e_0)$ such that $p \circ \tilde{\psi} = \psi$.

Hint. For every point $y \in X$ there is a path from x_0 to y and it can be lifted. But does this define $\tilde{\psi}(y)$ uniquely? Is the result continuous?

Saved

Bonus



Q4 Image/PDF question

If G and H are groups, we define a multiplication on G imes H by $(g_1,h_1)(g_2,h_2) = (g_1g_2,h_1h_2).$

A. (5 points) Verify that G imes H is again a group.

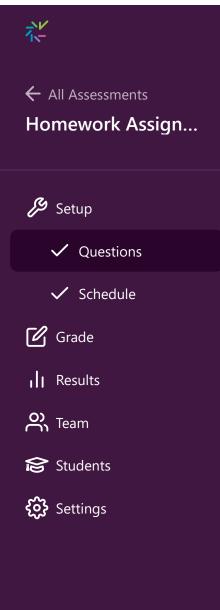
B. (10 points) If (X, x_0) and (y, y_0) are based spaces, we let $(X, x_0) \times (Y, y_0)$ be the based space $(X \times Y, (x_o, y_0))$. Show that $\pi_1((X, x_0) \times (Y, y_0)) \simeq \pi_1(X, x_0) \times \pi_1(Y, y_0)$. (People often ignore basepoints and write $\pi_1(X \times Y) = \pi_1(X) \times \pi_1(Y)$, but that's a bit less accurate).

Q5 Image/PDF question

10 points

Let 8 be the space that looks like the numeral 8, with the basepoint in the centre. Use the "Mexican cross" covering of 8 to show that $\pi_1(8)$ is equal, as a set, to the set of words of the form $a^{\alpha_1}b^{\beta_1}a^{\alpha_2}b^{\beta_2}\cdots a^{\alpha_n}b^{\beta_n}$, where *n* is a positive integer and α_i and β_i are non-zero integers for all *i*, except that α_1 is allowed to be 0 and β_n is allowed to be 0. (For simplicity we ignore the group structure on $\pi_1(8)$ here).

O Preview



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Setup complete

You can begin grading after the due date