Homework Assignment 9

Solve and submit your solutions of the following problems. Note that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Due date

Tuesday, November 26, 2024 11:59 pm (Eastern Standard Time)

Late penalty

5% deducted per hour

Q1 (0 points)

Read sections 51-55 in Munkres' textbook (Topology, 2nd edition). Remember that reading math isn't like reading a novel! If you read a novel and miss a few details most likely you'll still understand the novel. But if you miss a few details in a math text, often you'll miss everything that follows. So reading math takes reading and rereading and rerereading and a lot of thought about what you've read.

Q2 (10 points)

Show that the two definitions given in class for a covering $p : E \to B$ are equivalent:

Definition 1. There is an open cover $\mathcal{U}$ of B such that for every $U \in \mathcal{U}$ there is a discrete set D and a homeomorphism $\phi : U \times D \to p^{-1}(U)$ such that $p \circ \phi = \pi_U$, where $\pi_U : U \times D \to U$ is the projection on the first component.

Definition 2. There is an open cover $\mathcal{U}$ of B such that for every $U \in \mathcal{U}$, its inverse image $p^{-1}(U)$ is a union of disjoint open sets U_β in E such that for each β the restriction of p to U_β is a homeomorphism of U_β with U.

Q3 (20 points)

A space X is called "locally path connected" if for every $x \in X$ and every open set $U \subset X$ with $x \in U$, there is a path-connected open set V such that $x \in V \subset U$.

Show that if $p : (E, e_0) \to (B, b_0)$ is a covering, if (X, x_0) is path connected, locally path connected, and simply connected and if $\psi : (X, x_0) \to (B, b_0)$ is given, then there is a unique $\tilde{\psi} : (X, x_0) \to (B, b_0)$ such that $p \circ \tilde{\psi} = \psi$.

Hint. For every point $y \in X$ there is a path from x_0 to y and it can be lifted. But does this define $\tilde{\psi}$ uniquely? Is the result continuous?

Q4 (15 points)

If G and H are groups, we define a multiplication on $G \times H$ by $(g_1, h_1)(g_2, h_2) = (g_1 g_2, h_1 h_2)$.

A. (5 points) Verify that $G \times H$ is again a group.

B. (10 points) If (X, x_0) and (y, y_0) are based spaces, we let $(X, x_0) \times (Y, y_0)$ be the based space $(X \times Y, (x_o, y_0))$. Show that $\pi_1((X, x_0) \times (Y, y_0)) \simeq \pi_1(X, x_0) \times \pi_1(Y, y_0)$. (People often ignore basepoints and write $\pi_1(X \times Y) = \pi_1(X) \times \pi_1(Y)$, but that's a bit less accurate).

Q5 (10 points)

Let 8 be the space that looks like the numeral 8, with the basepoint in the centre. Use the "Mexican cross" covering of 8 to show that $\pi_1(8)$ is equal, as a set, to the set of words of the form $a^{\alpha_1} b^{\beta_1} a^{\alpha_2} b^{\beta_2} \cdots a^{\alpha_n} b^{\beta_n}$, where n is a positive integer and α_i and β_i are non-zero integers for all i, except that α_1 is allowed to be 0 and β_n is allowed to be 0. (For simplicity we ignore the group structure on $\pi_1(8)$ here).

Ready to submit?

- Please ensure all pages are in order and rotated correctly before you submit
- You will not be able to resubmit your work after the due date has passed.

Please wait...