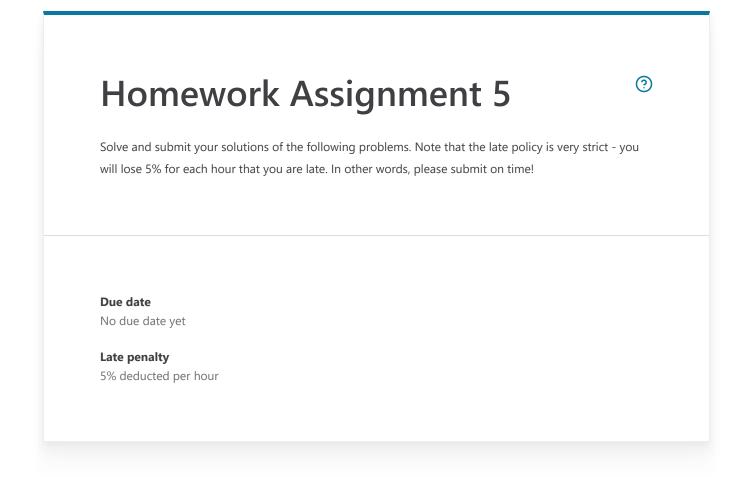
This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.



Q1 (0 points)

Read sections 19 through 22 in Munkres' textbook (Topology, 2nd edition). Remember that reading math isn't like reading a novel! If you read a novel and miss a few details most likely you'll still understand the novel. But if you miss a few details in a math text, often you'll miss everything that follows. So reading math takes reading and rereading and rerereading and a lot of thought about what you've read. Also, preread sections 23 and 24, just to get a feel for the future.

Q2 (10 points)

(Munkres pp 118 ex 6)

Let x_1, x_2, \ldots be a sequence of points in a product space $\prod X_{\alpha}$. Show that this sequence converges to the point x iff $\pi_{\alpha}(x_k) \to \pi_{\alpha}(x)$ for every α . (Note that if we don't specify the topology on a product space, we take it to be the cylinders topology).

Is the same fact true in the box topology?

Q3 (10 points)

(Munkres pp 118 ex 7)

Let \mathbb{R}^{∞} be the subset of $\mathbb{R}^{\mathbb{N}}$ consisting of the sequences that are almost always 0 - meaning, that are not zero only for finitely many indices. What is the closure of \mathbb{R}^{∞} in $\mathbb{R}^{\mathbb{N}}$ in the box and in the cylinders topology?

(Whatever is your answer, you need to prove it, of course).

Q4 (10 points)

(Munkres pp 126 ex 2)

Show that $\mathbb{R} \times \mathbb{R}$ in the dictionary order topology is metrizable.

Q5 (10 points)

(Munkres pp 126 ex 3, modified)

Let X be a metric space with metric d.

(a) Show that $d \colon X imes X o \mathbb{R}$ is continuous.

(b) Show that the metric topology on X is the weakest (coarsest, smallest) topology on X relative to which $d: X \times X \to \mathbb{R}$ is continuous.

Q6 (10 points)

(Munkres pp 127 ex 5)

With the same notation as in Q3, what is the closure of \mathbb{R}^{∞} in $\mathbb{R}^{\mathbb{N}}$ using the uniform topology, defined by the metric $d(x, y) = \sup_k (\min(1, |x_k - y_k|)).$

Q7 (10 points)

(Munkres pp 127 ex 6)

With the same "uniform metric" as in the previous question, with some fixed $x = (x_1, x_2, ...)$ in $\mathbb{R}^{\mathbb{N}}$, and with some 0 < r < 1, let $U(x, r) = \prod_k (x_k - r, x_k + r)$ (a product of intervals). Show that

(a) U(x,r) is not equal to the ball $B_r(x)$.

(b) U(x,r) is not even open in the uniform topology.

(c) $B_r(x) = igcup_{s < r} U(x,s).$

