Homework Assignment 3

Solve and submit your solutions of the following problems. Note that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Due date
Tuesday, September 24, 2024 11:59 pm (Eastern Daylight Time)

Late penalty
5% deducted per hour

Q1 (0 points)

Read sections 14 through 17 in Munkres' textbook (Topology, 2nd edition). Remember that reading math isn't like reading a novel! If you read a novel and miss a few details most likely you'll still understand the novel. But if you miss a few details in a math text, often you'll miss everything that follows. So reading math takes reading and rereading and rerereading and a lot of thought about what you've read. Also, preread sections 18 through 22, just to get a feel for the future.

Q2 (10 points)

(Munkres pp 92 ex 4)

A map (meaning, function) $f: X \to Y$ is said to be an *open map* if for every open set U in X, the set $f(U)$ is open in Y. Show that the projection maps $\pi_X: X \times Y \to X$ and $\pi_Y: X \times Y \to Y$ are open.

Q3 (10 points)

(Munkres pp 92 ex 8)

If L is a straight line in the plane, describe the topology L inherits as a subspace of $\mathbb{R}_\ell \times \mathbb{R}$ and as a subspace of $\mathbb{R}_\ell \times \mathbb{R}_\ell$. In each case it is a familiar topology!

Note 1. There are several cases to consider, depending on the direction of L.

Note 2. One should think that "describe" for verbal things is like "simplify" for formula-things. The topologies in question were given by a verbal description; the content of the question is that you should be giving a simpler one, and the best is if it is of the form "the topology in question is the trivial topology", or something like that.

Q4 (10 points)

(Munkres pp 92 ex 9)

Show that the dictionary order topology on $\mathbb{R} \times \mathbb{R}$ is the same as the product topology $\mathbb{R}_d \times \mathbb{R}$, where $\mathbb{R}_d$ is $\mathbb{R}$ with its discrete topology. Compare this topology with the standard topology on $\mathbb{R}^2$.

Q5 (0 points)

Challenge Problem (not for credit). Let X and Y be topological spaces and let $A \subset X$ and $B \subset Y$ be subsets thereof. Using only the definitions in terms of continuity of certain functions, show that the topology induced on $A \times B$ as a subset of the product $X \times Y$ is equal to the topology induced on it as a product of subsets of X and of Y. You are allowed to use the fact that two topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on some set W are equal if and only if the identity map regarded as a map from $(W, \mathcal{T}_1)$ to $(W, \mathcal{T}_2)$ is a continuous and its inverse is also continuous. Notions like "open sets" and "basis for a topology" are **not allowed** in your proof.

Ready to submit?

- Please ensure all pages are in order and rotated correctly before you submit
- You will not be able to resubmit your work after the due date has passed

Please wait...