This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

Homework Assignment 2

Solve and submit your solutions of the following problems. Note that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Due date

Tuesday, September 17, 2024 11:59 pm (Eastern Daylight Time)

Late penalty

5% deducted per hour

Q1 (0 points)

Find some "cup is homeomorphic to a donut" videos online and watch them.

Also, make your own, if you are so inclined! Submit it to be added to our photo album!

Q2 (10 points)

(Munkres pp 83 ex 1)

Let X be a topological space, and let A be a subset of X such that for every $x \in A$ there is an open set U with $x \in U \subset A$. Prove that A is open.

Q3 (0 points)

(Munkres pp 83 ex 3)

(a) Let X be a set. Show that the "countable complement topology" on X is a topology. Namely, show that $\mathcal{T}_c := \{\emptyset\} \cup \{U \subset X : X - U \text{ is countable}\}$ is a topology on X.

(b) Let X be an infinite set. Is the collection $\mathcal{T}_\infty := \{X\} \cup \{U \subset X : X - U \text{ is infinite}\}$ a topology on X?

Q4 (0 points)

(Munkres pp 83 ex 4)

(a) If $\{\mathcal{T}_\alpha\}$ is a family of topologies on X, show that $\bigcap_\alpha \mathcal{T}_\alpha$ is also a topology on X.

(b) If $\{\mathcal{T}_\alpha\}$ is a family of topologies on X, show that there is a unique smallest topology on X which contains all the $\mathcal{T}_\alpha$'s, and a unique largest topology on X contained in all the $\mathcal{T}_\alpha$'s.

(c) With $X = \{a, b, c\}$, $\mathcal{T}_1 = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\mathcal{T}_1 = \{\emptyset, X, \{a\}, \{b, c\}\}$, find the smallest topology containing $\mathcal{T}_1$ and $\mathcal{T}_2$ and the largest topology contained in $\mathcal{T}_1$ and in $\mathcal{T}_2$.

Q5 (0 points)

(Munkres pp 83-84 ex 8)

(a) Show that the countable collection $\{(a,b) : a,b \in \mathbb{Q},\ a<b\}$ is a basis for a topology on $\mathbb{R}$ and that it generates the standard topology on $\mathbb{R}$.

(b) Show that the countable collection $\{[a,b) : a,b \in \mathbb{Q},\ a<b\}$ is a basis for a topology on $\mathbb{R}$ yet the topology it generates is different from the lower limit topology on $\mathbb{R}$.

Q6 (10 points)

Prove that a function $f\colon X \to Y$ is continuous, where both X and Y are taken with the finite-complement ("fc") topology, if and only if it is constant or finite-to-one. ("Finite-to-one" means that any $y \in Y$ has at most finitely many inverse images: $\forall y \in Y\ |f^{-1}(\{y\})| < \infty$).

Ready to submit?

- Please ensure all pages are in order and rotated correctly before you submit
- You will not be able to resubmit your work after the due date has passed

Please wait...