X_n w/ metric d_n bndd by 1.

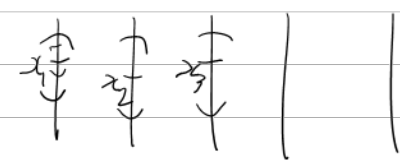
$X = \prod X_n \qquad d(x,y) = \sup_n \frac{1}{n} d_n(x_n, y_n)$

$\mathcal{T}_{cyl} \subset \mathcal{T}_{met}$ NTS every cylinder is open in the metric top.

Pick a cylinder. $C = \prod_{n=1}^{N} U_n \times \prod_{n=N+1}^{\infty} X_n$ where $U_n \subset X_n$ is open

Let $x = (x_n) \in C$; $x_n \in U_n$ for $n = 1 \dots N$

Find $\epsilon_n > 0$ $(n \leq N)$ s.t. $B_{\epsilon_n}(x_n) \subset U_n$



set. $\epsilon = \min_{n \leq N} \frac{1}{n} \epsilon_n$

Claim $B_\epsilon(x) \subset C$ Indeed, pick $y \in B_\epsilon(x)$, meaning $y = (y_n)$ &

$\frac{1}{n} d_n(x_n, y_n) < \epsilon \leq \frac{1}{n} \epsilon_n$ so $d_n(x_n, y_n) < \epsilon_n$ so $y_n \in B_{\epsilon_n}(x_n) \subset U_n$

... $\square$

$\mathcal{T}_{met} \subset \mathcal{T}_{cyl}$ NTS every metric ball is contained in a cylinder

Let $x \in X$ & $\epsilon > 0$

$B_\epsilon(x) = \{ (y_n) = y : d(x,y) < \epsilon \}$

$= \{ y = (y_n) : \sup \frac{1}{n} d_n(x_n, y_n) < \epsilon \}$

$= \{ y = (y_n) : \max_{n \leq \epsilon^{-1}} \frac{1}{n} d_n(x_n, y_n) < \epsilon \}$

$= \{ y = (y_n) : \forall n < \epsilon^{-1} \; d_n(x_n, y_n) < \epsilon n \}$

$= U_1 \times \dots \times U_{\lceil \epsilon^{-1} \rceil} \times X_{\lceil \epsilon^{-1} \rceil + 1} \times \dots$

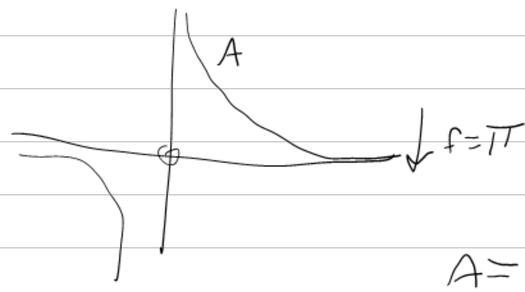
where $U_n = B_{\epsilon n}(x_n)$.

$X = \mathbb{R}^{\mathbb{R}}$ $\quad B = \{f \in X : |\operatorname{supp} f| < \infty\}$

1. $T \in \overline{B}$ $\quad$ 2. $T \notin$ sq-cl B

Pf of 2

$f(\overline{A}) \subset \overline{f(A)}$



$A = \{(x, y) : x \cdot y = 1\}$

$m : \mathbb{R}^2 \to \mathbb{R}$ $\quad m(x, y) = xy$

is cont.

$A = m^{-1}(\{1\})$ is closed.