Homework Assignment 10

Due: Thursday, December 9, 2021 11:59 pm (Eastern Standard Time)

Assignment description

Solve and submit your solutions of the following problems. They are all taken from Munkres' *Analysis on Manifolds*, page 151. Note the unusual due day - Thursday rather than Friday! Note also that the late policy remains strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (10 points)

If $V = \{(x,y,z)\colon x^2+y^2+z^2<a^2 \text{ and } z>0\}$, use the spherical coordinate transformation $g(r,\phi,\theta) = (r\cos\phi\cos\theta, r\cos\phi\sin\theta, r\sin\phi)$ to express $\int_V z$ as an integral over an appropriate set in $\mathbb{R}^3_{r,\phi,\theta}$.

Q2 (10 points)

Let $f(x,y) = 1/(x^2+y^2)$. Determine if f is integrable over $U_1 = \{(x,y)\colon 0<x^2+y^2<1\}$ and over $U_2 = \{(x,y)\colon x^2+y^2>1\}$.

Q3 (10 points)

Let $B = \{(x,y)\colon x>0,\ y>0,\ 1<xy<2,\ x<y<4x\}$. Compute $\int_B x^2y^3$. Hint: Set $x = u/v$ and $y = uv$.

Q4 (10 points)

Let T be the tetrahedron in $\mathbb{R}^3$ having vertices $(0,0,0)$, $(1,2,3)$, $(0,1,2)$, and $(-1,1,1)$. Compute $\int_T f$, where $f(x,y,z) = x + 2y - z$. Hint: use a linear transformation to change variables.

Q5 (10 points)

Let $0 < a < b$, and let T be a solid torus, the result of spinning around the z axis the disk of radius a around the point $(b,0,0)$ in the xz-plane. What is the volume of T? Hint: T is the image under $g(r,\theta,z) = (r\cos\theta, r\sin\theta, z)$ of $A = \{(r,\theta,z)\colon (r-b)^2 + z^2 \le a^2 \text{ and } 0 \le \theta < 2\pi\}$.