Homework Assignment 8

Due: Friday, November 26, 2021 11:59 pm (Eastern Standard Time)

Assignment description

Solve and submit your solutions of the following problems. They are taken from Spivak's book, pages 56 and 61-62. The late policy remains strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Sorry for being nearly 10 hours late in assigning this assignment.

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (10 points)

Recall from reading the text that a set C is called "Jordan-measurable" if it is bounded and its boundary has measure 0, or equivalently, if it is bounded and its characteristic function is integrable in some rectangle that contains C.

Spivak's 3-22. If A is a Jordan-measurable set and $\epsilon > 0$, show that there is a compact Jordan-measurable set $C \subset A$ such that the volume of $A \setminus C$ is less than ϵ.

Q2 (10 points)

Recall from page 26 of the text that if f is a real-valued function, then $D_i f$ denotes its ith partial derivative, and $D_{i,j} f$ denotes the jth partial derivative of its ith partial derivative:

$$D_{i,j} f = D_j(D_i f)$$

(assuming all these quantities exist).

Spivak's 3-28. Use Fubini's theorem to give an easy proof that $D_{1,2} f = D_{2,1} f$, if these are continuous (hint in text).

Q3 (10 points)

Spivak's 3-32. Let $f\colon [a,b]\times[c,d]\to\mathbb{R}$ be continuous and suppose $D_2 f$ is continuous. Define $F(y)=\int_a^b f(x,y)dx$. Prove *Leibnitz' rule:* $F'(y)=\int_a^b D_2 f(x,y)dx$. (Hint in text).

Q4 (10 points)

Spivak's 3-34 (modified). Let $g_1, g_2\colon \mathbb{R}^2\to\mathbb{R}$ be continuously differentiable and suppose $D_1 g_2 = D_2 g_1$. Let

$$f(x,y)=\int_0^x g_1(t,0)dt+\int_0^y g_2(x,t)dt.$$

Show that $D_1 f = g_1$ and $D_2 f = g_2$. (Hint: Use the previous question).