Homework Assignment 5

Due: Friday, October 22, 2021 11:59 pm (Eastern Daylight Time)

Assignment description

Solve and submit your solutions of the following problems. They are taken from Spivak's book, pages 39 and 40. Note that the late policy remains strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (10 points)

Spivak's 2-36. Let $A \subset \mathbb{R}^n$ be an open set and $f: A \to \mathbb{R}^n$ a continuously diffable 1-1 function such that $f'(x)$ is invertible for all $x \in A$. Show that $f(A)$ is an open set and that $f^{-1}: f(A) \to A$ is diffable. Show also that $f(B)$ is open for any open set $B \subset A$.

Q2 (10 points)

Spivak's 2-37. (a) Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a continuously diffable function. Show that f is *not* 1-1 (hint in textbook).

It is not required to submit part (b) of this question (though if you submit it, you will not lose any points, of course).

(b) Generalize this result to the case of a continuously diffable $f: \mathbb{R}^n \to \mathbb{R}^m$, where $n > m$.

Q3 (10 points)

Spivak's 2-38. (a) If $f: \mathbb{R} \to \mathbb{R}$ satisfies $f'(a) \neq 0$ for all $a \in \mathbb{R}$, show that f is 1-1 on $\mathbb{R}$.

(b) Define $f: \mathbb{R}^2 \to \mathbb{R}^2$ by $f(x, y) = (e^x \cos y, e^x \sin y)$. Show that $f'(x, y)$ is always invertible yet f is not 1-1.