This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

Homework Assignment 2

Due: Friday, October 1, 2021 11:59 pm (Eastern Daylight Time)

Assignment description

Solve and submit your solutions of the following questions. Note that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (10 points)

Spivak's 1-16. Find the interior, exterior, and boundary of the sets

$$A_1 = \{x \in \mathbb{R}^n : |x| \leq 1\},$$

$$A_2 = \{x \in \mathbb{R}^n : |x| = 1\},$$

$$A_3 = \{x \in \mathbb{R}^n : \forall i \; x_i \in \mathbb{Q}\}.$$

Q2 (10 points)

Spivak's 1-21. (a) If A is closed and $x \notin A$, prove that there is a number $d > 0$ such that $|y - x| \geq d$ for all $y \in A$.

(b) If A is closed, B is compact and $A \cap B = \emptyset$, prove that there is $d > 0$ such that $|y - x| \geq d$ for all $x \in A$ and $y \in B$ (hint available in textbook).

(c) Give a counterexample in $\mathbb{R}^2$ if A and B are closed but neither is compact.

Q3 (0 points)

Spivak's 1-22. If U is open and $C \subset U$ is compact, show that there is a compact set $D \subset U$ whose interior contains C.

Q4 (0 points)

Spivak's 1-25. Prove that a linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ is continuous (hint available in textbook).

Q5 (10 points)

Spivak's 1-26, rephrased. Let $A = \{(x, y) \in \mathbb{R}^2 : x > 0 \text{ and } 0 < y < x^2\}$. Let $f = 1_A : \mathbb{R}^2 \to \mathbb{R}$ be the *indicator* function of A, defined by $f(x, y) = 1$ if $(x, y) \in A$, and $f(x, y) = 0$ otherwise. Show that f is not continuous at $(0, 0)$, yet its restriction to every straight line through $(0, 0)$ is continuous at $(0, 0)$.

Q6 (0 points)

Spivak's 1-28. If $A \subset \mathbb{R}^n$ is not closed, show that there is a continuous function $f: A \to \mathbb{R}$ which is unbounded (hint available in textbook).

Q7 (10 points)

For submission. Prove that a set C is compact if and only if every open cover $\mathcal{U}$ of C that is closed under unions of pairs (namely, $(A \in \mathcal{U}) \text{ and } (B \in \mathcal{U}) \Longrightarrow (A \cup B \in \mathcal{U})$) has a set T such that $C \subset T$.

Do not submit, yet ponder. Okay, so we have a statement that can serve as an alternative definition of compactness. Perhaps it is better? Go over all the theorems about compactness shown in class and re-derive them with the definition of compactness replaced with the alternative one. Have they become easier or harder? Less or more "natural"?