This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

# Term Test 2

**Due:** Tuesday, January 18, 2022 7:30 pm (Eastern Standard Time)

## Assignment description

**Solve** all 5 problems on this test, and do Task 6.

Each problem is worth 20 points.

You have two hours to write this test, and another 25 minutes for Task 6 and for uploading.

**Allowed material.** Open book(s), open notes, but you can only use the internet (during the exam) to read the exam, to submit the exam, and to connect with the instructor/TAs to ask clarification questions. No contact allowed with other students or with any external advisors, online or in person.

**Neatness counts! Language counts!** The *ideal* written solution to a problem looks like a page from a textbook; neat and clean and consisting of complete and grammatical sentences. Definitely phrases like "there exists" or "for every" cannot be skipped. Lectures are mostly made of spoken words, and so the blackboard part of proofs given during lectures often omits or shortens key phrases. The ideal written solution to a problem does not do that.

## Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

### Q1 (20 points)

Directly from the definition, show that the set $\{(t,t)\colon t\in\mathbb{Q}\cap[0,1]\}\subset\mathbb{R}^2$ is of content 0 in $\mathbb{R}^2$.

**Tip.** Don't start working! Read the whole test first. You may wish to start with the questions that are easiest for you.

### Q2 (20 points)

Show that every open set in $\mathbb{R}^n$ is the union of countably many compact sets.

**Tip.** In math exams, "show" means "prove".

### Q3 (20 points)

Compute $\int_{x^2+y^2\leq R^2}\frac{dx\,dy}{1+x^2+y^2}$.

**Tip.** Explain every step of your computation!

**Note added 5:34pm.** $R$ is a positive real number.

## Q4 (20 points)

A bounded non-negative function $f$ is continuous on $\mathbb{R}^n$ except for a set of measure 0, and it is known that there is a constant $M$ such that $\int_R f \leq M$ for every rectangle $R$ in $\mathbb{R}^n$. Show that $f$ is integrable (NT) on $\mathbb{R}^n$

**Tip.** Remember that the definition of integrability (NT) starts with a PO1. You can't prove anything about integrability (NT) unless you start the same way.

## Q5 (20 points)

A continuously differentiable map $g\colon \mathbb{R}^n \to \mathbb{R}^n$ is called "volume-preserving" if for every Jordan-measurable set $B$, the set $g^{-1}(B)$ is also Jordan measurable and its volume is equal to the volume of $B$. Show that the function $h\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $h(x,y) = (y, x+y)$ is volume-preserving.

**Tip.** Once you have finished writing a test, if you have time left, it is always a good idea to go back and re-read and improve everything you have written, and perhaps even completely rewrite any parts that came out messy.

## Task 6 (0 points)

Please copy in your own handwriting, fill in the missing details, and sign the statement below, and then submit it along with a photo of your student ID card to complete this test. (See a sample submission below)

By signing this statement, I am attesting to the fact that I, [name], [student number], have abided fully to the Code of Behaviour on Academic Matters. I have not committed academic misconduct, and I am aware of the penalties that may be imposed if I have committed an academic offence.

Signature: