$$\lim_{\alpha\to 0} \frac{L(\underbrace{\alpha v}_{h})}{|\underbrace{\alpha v}_{h}|} \qquad \frac{L(h)}{|h|} \longrightarrow 0$$

$$\frac{L(v)}{|v|} = \lim = \ldots = 0$$

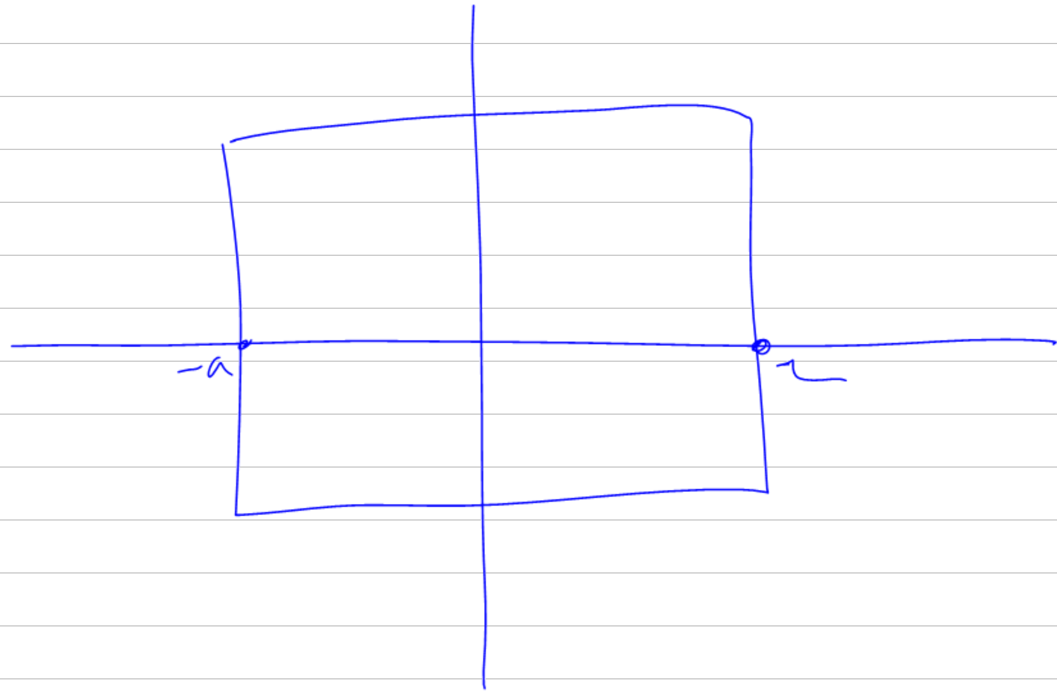
---

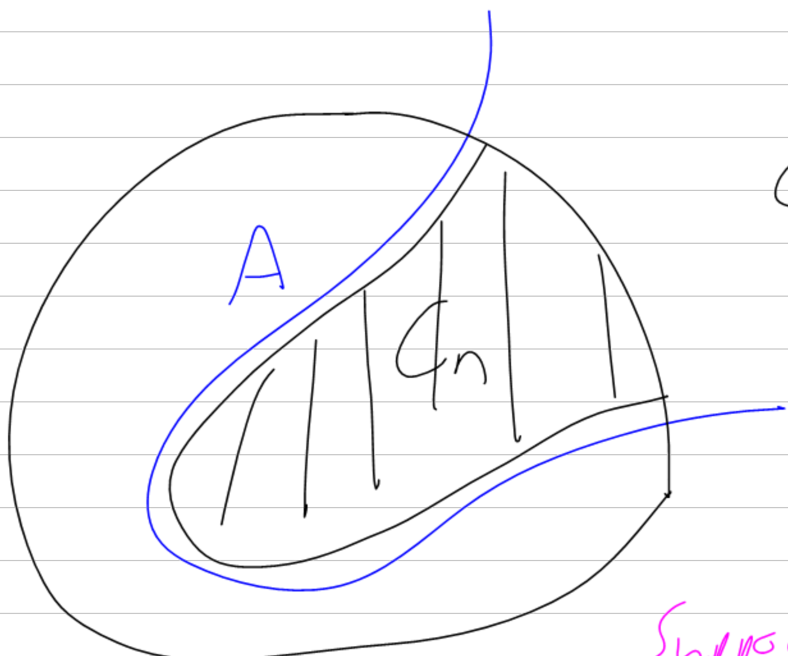
$$(f^{-1})'(y) = \left[f'(f^{-1}(y))\right]^{-1} \qquad g = f^{-1}$$

$$g'(y) = f'(g(y))^{-1} \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial g_1}{\partial y_1} & \cdots & \frac{\partial g_1}{\partial y_n} \\ \vdots & \frac{\partial g_2}{\partial y_2} & \\ \frac{\partial g_n}{\partial y_1} & \cdots & \frac{\partial g_n}{\partial y_n} \end{pmatrix} = \begin{pmatrix} \text{a messy explicit formula involving } +,-,\times,\div \\ \text{\& the partials} \end{pmatrix}$$

of f evaluated at $g(y)$




$$C_n = \left\{ x \in A : \begin{array}{l} |x| \le n \\ d(x, A^c) \ge \frac{1}{n} \end{array} \right\}$$

1. $C_n$ is compact.
2. $\bigcup C_n \subseteq A$, $\supseteq$

Suppose $y \in A$, $|y| < \infty$, $d(y, A^c) > 0$

$f$ cont except meas 0 & loc bdd.

$A = \bigcup A_n$ $A$ open, $A_n$ open, $A_n \subset A_{n+1}$

$\exists M$ s.t. $\int_{A_n} |f| < M \iff \lim_{n\to\infty} \int_{A_n} |f| < \infty$

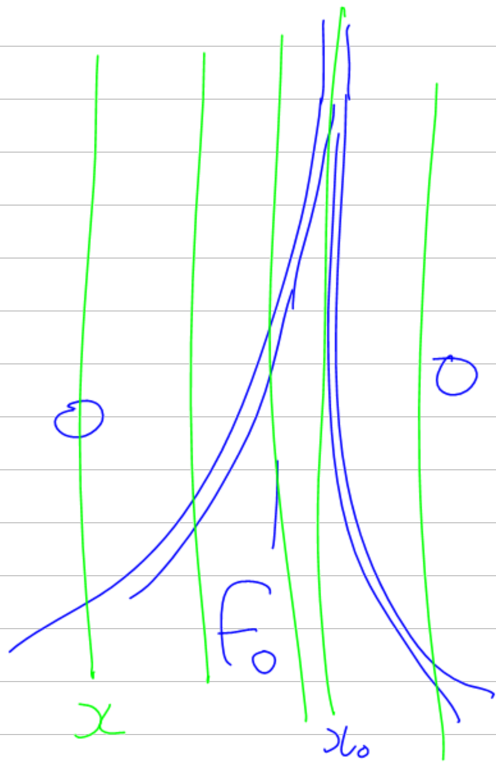
$\Rightarrow f$ is integrable

$$\sum_{i=1}^{N} \int \varphi_i |f| = \int \underbrace{\left(\sum_{1}^{N} \varphi_i\right)}_{\text{cmpct.}} |f| = \int_{A_n} \left(\sum_{1}^{N} \varphi_i\right) |f|$$

$\subset A_n$ for $n \gg 1$.

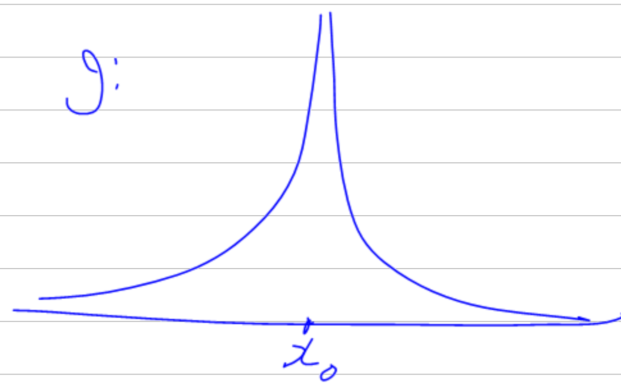
$$\mathbb{Q} \cap [0,1] = \{r_i\}_{i=1}^{\infty}$$

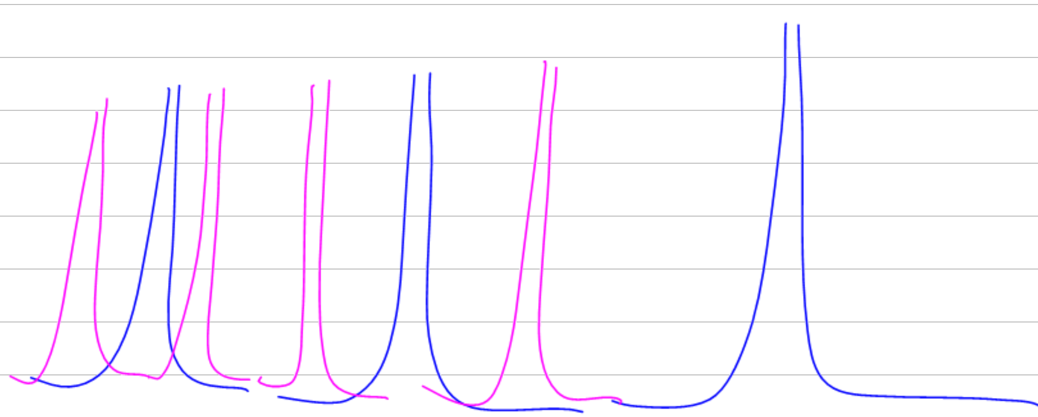
$$A = \bigcup \left(r_i - \frac{0.01}{2^n}, r_i + \frac{0.01}{2^n}\right) \cup (2,3)$$




$$g(x) = \int f(x,y)\,dy$$

$g$:



$$F(x,y)=\sum_{k=1}^{\infty}\frac{1}{k!}f_0(x-r_k,y)$$

$$\{r_k\}=\mathbb{Q}$$

$$g(x)=\int F(x,y)\,dy \qquad g(r)=\cancel{\infty}\text{ undef}$$

$$r\in\mathbb{Q}$$

---

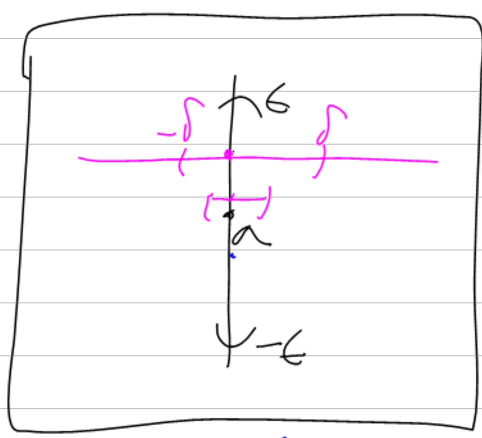
$F:\mathbb{R}^2\to\mathbb{R}$ cont. $\quad F(a)>0$, $\quad a\in\mathbb{R}^2$

$$F^{-1}((0,\infty))=U \qquad U \text{ is open}$$

$(0,\infty)$ open

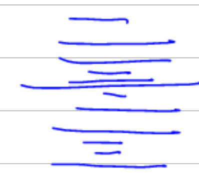
$a\in U$

$F|_U>0$



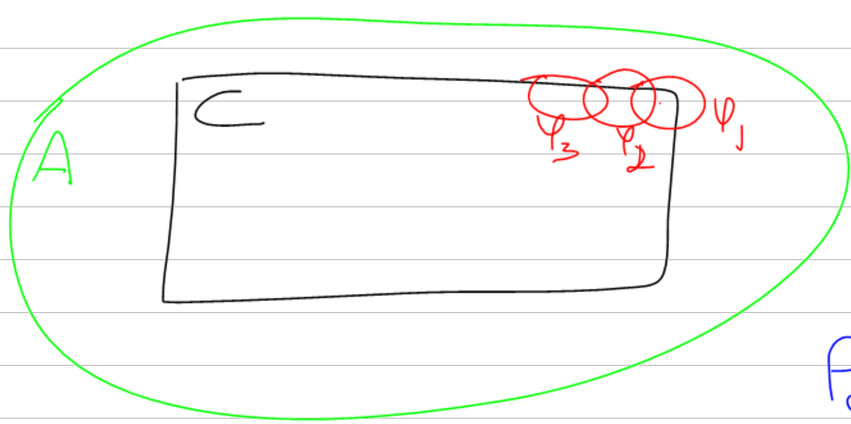

$\xrightarrow{F}\mathbb{R}$

A :



$a=(x_0,y_0)$

We find $A\ni a$, $F|_A>0$ and $\exists\,\epsilon>0$ $\forall y$ $|y-y_0|<\epsilon$ $\exists\,\delta>0$ if $|x-x_0|<\delta$ then $(x,y)\in A$. Is A open? No.




$$F\big|_C = 1$$

$$\text{supp}\, F \subset A$$

$$F_0 = \sum_{i=1}^{N} \varphi_i$$


1. $F_0\big|_C \geq 1$
2. $\text{supp}\, F_0 \subset A$

$$F = g \circ F_0$$

---

Spivak 3-40 page 73 is False?

$$g : \mathbb{R}^2 \to \mathbb{R}^2$$

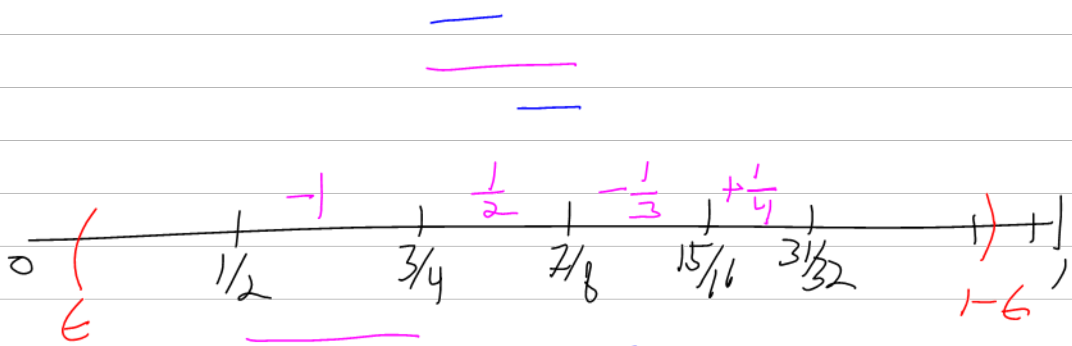
$$g\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \end{pmatrix} \qquad g' = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

not diagonal

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \xrightarrow{g_1} \begin{pmatrix} y_1 \\ x_2 \end{pmatrix} \xrightarrow{g_2} \begin{pmatrix} x_1 + x_2 = y_1 \\ x_1 - x_2 = y_2 \end{pmatrix}$$

$$g_1\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 \end{pmatrix} \qquad g_2\begin{pmatrix} y_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_1 - 2x_2 \end{pmatrix}$$

Spivak 3-37b page 66




$$\lim_{\epsilon \to 0} \int_{\epsilon}^{1-\epsilon} f = \sum \frac{(-1)^n}{n} = \log 2$$

$$\log(1+x) = \sum_{n>0} (-1)^{n+1} \frac{x^n}{n}$$

---

$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x \\ e^x + y \end{pmatrix}$$

$$\mathcal{T}^0(V) = \mathbb{R}$$

$$\mathcal{T}^k(V) = \{\text{m.l. } V^k \longrightarrow \mathbb{R}\}$$

$V^k$ = sequences of length $k$ of elements of $V$

$V^0 = $ seqs of len $= 0$ $(\cancel{v_1 \ldots v_k})$

$= \{ ( ) \}$

$$\mathcal{T}^0(V) = \{ t : \{()\} \longrightarrow \mathbb{R} \} \sim \mathbb{R}$$

---

$f : \mathbb{R}^2_{x,y} \longrightarrow \mathbb{R}$ cont. & integrable & bndd

$f \geq 0$

$\varphi_i$ for $\mathbb{R}$ s.t. supp $\varphi_i$ is cmpct. then $\varphi_i(x)\varphi_j(y)$ is

a PO 1 for $\mathbb{R}^2$

$$\sum_{i,j} \int \varphi_i(x)\varphi_j(y) f(x,y)\, dx\, dy$$

$$\underset{\text{Fub}}{\overset{\text{OT}}{=}} \sum_{i,j} \int dx\, \varphi_i(x) \int dy\, \varphi_j(y) f(x,y)\, dy$$

$$\sum_{i,j} \varphi_i(x)\varphi_j(y) = \Big(\sum_i \varphi_i(x)\Big)\Big(\sum_j \varphi_j(y)\Big)$$

$$\begin{matrix} & \parallel & \\ 1 & \cdot & 1 = 1 \end{matrix}$$

TT3 Q2

$$\phi\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix} e^x\cos y \\ e^x \sin y\end{pmatrix} \qquad \phi' = \begin{pmatrix} \ & \ \\ \ & \ \end{pmatrix}$$

$$\phi_*(\xi) = \left(\phi(p), \phi'(p)\cdot v\right)$$

$$\xi = (p, v)$$