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MAT257 Term Test 3 Information and Rejected Questions

- The test will take place on Tuesday March 9, 5-7PM (Toronto time), on Crowdmark (you will get a link by email about one minute before the official starting time). Other than documented accessibility matters, no exceptions!
- Our TAs Sebastian and Shuyang will hold extra pre-test office hours, in their usual zoom rooms. Sebastian on Monday 11-2 at Sebastian's Zoom (password vchat), and Shuyang on Friday and on Tuesday at 10:30-11:30 at Shuyang's Zoom (password vchat). These office hours replace some of their regular office hours; so Sebastian will not hold his regular office hours on March 15 and on March 22, and Shuyang will not hold her regular office hours on March 10 and 17.
- I will hold my regular office hours on Tuesday at 9-10 and 12-1, at `http://drorbn.net/vchat`.
- I will be available to answer questions throughout the exam, at my usual office (`http://drorbn.net/vchat`, but I'll add a waiting room). I will also be monitoring my regular email address (drorbn@math.toronto.edu) throughout the exam.
- There will be mishaps! I just hope that not too many. If you encounter one, document everything with specific details, times, and screen shots, and send me a message by Wednesday March 10 at 7PM. I will deal with these situations on a case by case basis.
- Don't let unanswered questions and/or mishaps paralyze you! If you need an answer but for whatever reason you cannot reach me, think hard, come up with what you think is the most reasonable answer/resolution, document as best as you can (for example, by adding a note on your submission), and act following your conclusions.
- Material: Everything up to and including Stokes' Theorem for Chains, with greater emphasis on the material that was not included in Term Test 2 (meaning, starting with the Baby Sard Theorem, and even more so, k-tensors and all that followed).
- Open book(s) and open notes but you can only use the internet (during the exam) to read the exam, to submit the exam, and to connect with the instructor to ask clarification questions. No contact allowed with other students or with any external advisors, online or in person.
- The format will be "Solve 7 of 7", or maybe "6 of 6" or "5 of 5".
- You will be required to copy in your handwriting and sign an academic integrity statement and submit it on Crowdmark along with the rest of your exam. If you wish, you may save time by preparing the academic integrity statement in advance as in this sample.
- You will be given an extra 20 minutes at the end of the exam to upload it and to copy/sign the academic integrity statement.
- The vast majority of students will do honest work, and I appreciate that. Out of respect for the honest students I will do my best to pursue and punish any cheating that may occur. I'm more experienced than you! If you plan to be dishonest, think again.
- To prepare: Do the TT3 "rejects" available below, but more important: make sure that you understand every single bit of class material so far!
- It is not the test I want! Class material and HW are important, but there won't be questions straight from class/HW. Many things in 2020/21 are not as we want them.

The following questions were a part of a question pool for the 2020-21 MAT257 Term Test 3, but at the end, they were not included.

1. Prove that the Change of Variables (COV) theorem holds even without the assumption on the invertibility of g'.

2. It is common to identify $\mathbb{R}^3$ with the space of column vectors of length 3, and to identify $(\mathbb{R}^3)^*$ with the space of row vectors of length 3. With this in mind, find the dual basis to the basis $v_1 = \begin{pmatrix}1\\3\\0\end{pmatrix}$, $v_2 = \begin{pmatrix}2\\4\\0\end{pmatrix}$, $v_3 = \begin{pmatrix}0\\0\\5\end{pmatrix}$ of $\mathbb{R}^3$.

3. Let V be a vector space, let $\phi\colon V \to V \times V$ be given by $\phi(v) = (v, v)$ and let $\psi\colon V \times V \to V \times V$ be given by $\psi(v, w) = (w, v)$. Let $B\colon V \times V \to \mathbb{R}$ be a bilinear function. Prove that $\phi^* B = 0$ iff $B + \psi^* B = 0$.

4. Prove that in S_k, for $k > 1$, there is an equal number of odd and even permutations.

5. Let $\sigma \in S_n$ be the permutation given by $\sigma i = i + 1$ for $i < n$ and $\sigma n = 1$. What is $\operatorname{sign}(\sigma)$?

6. Let $\phi\colon \mathbb{R}^2_{x,y} \to \mathbb{R}^2_{u,v}$ be given by $\phi(x, y) = (x^2 - y^2, 2xy)$. Compute $\phi^*(du \wedge dv)$ and $\phi_*\xi$, where ξ is the tangent vector to $\mathbb{R}^2_{x,y}$ given by $\xi = ((0, 1), (1, 0))$.

7. Let $\xi = (p, v)$ be a tangent vector to $\mathbb{R}^n$. Prove that there exists a path $\gamma\colon \mathbb{R} \to \mathbb{R}^n$ such that for every differentiable function $f\colon \mathbb{R}^n \to \mathbb{R}$ we have that $D_\xi f = (f \circ \gamma)'(0)$.

8. Let $\omega = \dfrac{xdy - ydx}{x^2 + y^2} \in \Omega^1\left(\mathbb{R}^2_{x,y} \setminus \{0\}\right)$, and let $f\colon Q = (0, \infty)_r \times [0, 2\pi]_\theta \to \mathbb{R}^2$ be given by $f(r, \theta) = (r\cos\theta, r\sin\theta)$.
   (a) Compute $f^*(\omega)$.
   (b) Show that ω is closed.
   (c) Show that $f^*(\omega)$ is exact on Q.
   (d) Show that ω is not exact on $\mathbb{R}^2_{x,y} \setminus \{0\}$.

9. Explain in detail how the vector-field operator grad arises as an instance of the exterior derivative operator $d\colon \Omega^k(\mathbb{R}^n) \to \Omega^{k+1}(\mathbb{R}^n)$, for some k and n.

10. Explain in detail how the vector-field operator div arises as an instance of the exterior derivative operator $d\colon \Omega^k(\mathbb{R}^n) \to \Omega^{k+1}(\mathbb{R}^n)$, for some k and n.

11. Let $f\colon [0, 1] \to \mathbb{R}$ be a differentiable function, and let $c\colon [0, 1]^2_{u,v} \to \mathbb{R}^2_{x,y}$ be the 2-cube given by $c(u, v) = (u, f(u)v)$. Use Stokes' theorem and the form $\omega = -ydx$ to show that $\int_c dx \wedge dy = \int_0^1 f(x)dx$. Can you interpret this result geometrically?

12. It is common to identify $\mathbb{R}^n$ with the space of column vectors of length n and to identify $(\mathbb{R}^n)^*$ with the space of row vectors of length n. Suppose $\phi \in (\mathbb{R}^m)^*$ is a row vector, and suppose $L\colon \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation presented relative to the standard bases of $\mathbb{R}^n$ and $\mathbb{R}^m$ by the matrix $A \in M_{m\times n}(\mathbb{R})$. Compute the row vector $L^*\phi$ (the pullback of ϕ via L).

Please watch this page for changes — I may add to it later. Last modified: Sunday 7th March, 2021, 16:45