# 240AlgebraI-141126 Hours 33-34: More Determinants

November-25-14 10:11 AM

Office Hour cancelled today; come instead to my talk at the Fields Institute at 2PM (prereqs: being <u>really</u> comfortable with uniform continuity & uniform convergence)

Basic properties of determinants:

0. $\det(I_n) = 1$

1. $\det\begin{pmatrix} \text{---} \\ \text{---}\ r_j\ \text{---} \\ \text{---}\ r_i\ \text{---} \\ \text{---} \end{pmatrix} = -\det\begin{pmatrix} \text{---}\ r_i\ \text{---} \\ \text{---} \\ \text{---}\ r_j\ \text{---} \\ \text{---} \end{pmatrix}$

2. $\det\begin{pmatrix} \text{---} \\ \text{---}\ cr\ \text{---} \\ \text{---} \end{pmatrix} = c\det\begin{pmatrix} \text{---} \\ \text{---}\ r\ \text{---} \\ \text{---} \end{pmatrix}$

3. $\det\begin{pmatrix} \text{---} \\ \text{---}\ r_i\ \text{---} \\ \text{---} \\ \text{---}\ r_j + cr_i\ \text{---} \\ \text{---} \end{pmatrix} = \det\begin{pmatrix} \text{---} \\ \text{---}\ r_i\ \text{---} \\ \text{---} \\ \text{---}\ r_j\ \text{---} \\ \text{---} \end{pmatrix}$

<u>Example</u> $\det\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, $\det\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

<u>Corollary</u> All that there is to know about determinants can be deduced from 0–3; also if $\det'$ satisfies 0–3, then $\det' = \det$.

<u>Thm</u> $A$ is invertible iff $\det(A) \neq 0$

<u>Thm</u> If $A = E_1 \cdots E_n$ is a product of elementary matrices, then $\det A = \det(E_1)\cdot\det(E_2)\cdots\det(E_n)$

<u>Claim</u> For square matrices, $AB$ invertible $\Leftrightarrow$ $A$ & $B$ are inv.

$\Leftarrow$ $(AB)^{-1} = B^{-1}A^{-1}$

$\Rightarrow$ $B(AB)^{-1}$ is a right inverse for $A$, & for square matrices if $AC = I$ then also $CA = I$

Was a HW problem

$\Rightarrow$ $B(AB)$ is a right inverse for $A$, & for square matrices, if $AC = I$ then also $CA = I$. problem

<u>Thm</u> $\det A \cdot B = \det A \cdot \det B$

<u>Thm</u> $\det A^T = \det A$

<u>Thm</u> Everything that's true for rows is also true for columns

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Skipped extras: 1. Other formulas for det. (row/col expansions, permutations)

2. A det formula for $A^{-1}$ & Kramer's law. 1. It's in all the books.

2. I've never used it in my life.

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... recall the formula for dets & sketch the proof of the basic properties:

$$|(a_{11})| := a_{11} \qquad \left|\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}\right| := \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \left|A_{\hat{1}\hat{j}}\right|$$

done line

Then prove:

1. Linear in the first row.
2. Multilinear in the rows.
3. Vanishes if the first two rows are equal.
4. Vanishes if two adjacent rows are equal.
5. Switches sign if two adjacent rows are interchanged.
6. Switches sign whenever two rows are interchanged.
7. $E^2_{i,c}$ & $E^2_{i,j,c}$ behaviour.