# Wednesday-4 AKT on 140129: More on swaddling and framing, $Z_{-1}$

January-28-14 9:06 AM

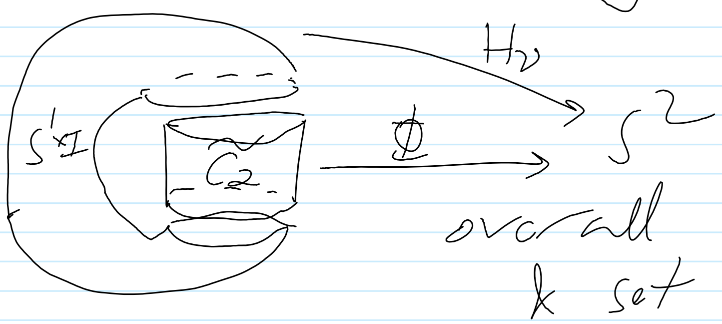
Goals: 1. Quickly finish swaddling/framing
2. start w/ proof of main Thm.

on board

$\eta(\gamma) := \int_{\tilde{C}_2(S^1)} \Phi^* w \qquad sl_2(\gamma, v) = l(\gamma, \gamma + \epsilon v)$

An alternative definition of $sl_2$:

a $v$ defines a homotopy $H_v: \dot\gamma \to -\dot\gamma$,




overall get $\Phi_v : T^2 \to S^2$
& set $sl_2 = \deg \Phi_v$

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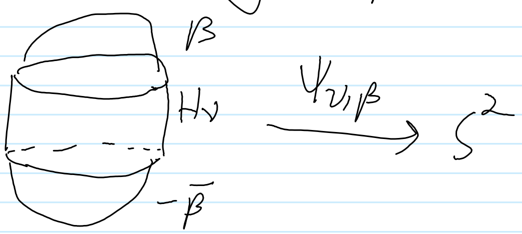
There is a pairing $\langle v, \beta \rangle \in \mathbb{Z}$ between framings and swaddling maps:




$\langle v, \beta \rangle = \deg \Psi_{v,\beta}$

<u>Thm</u> By declaring $\beta \leftrightarrow v \Leftrightarrow \langle v, \beta \rangle = 0$, there is a bijection between (homotopy classes of) swaddling maps and odd framings.

If $\beta \leftrightarrow v$, then $sl_1(\gamma, v) = sl_2(\gamma, \beta)$.

<u>Proof</u> HW.

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<u>Blatantly false theorem.</u>

D: $E_i(D)$: internal edges
$E_s(D)$: skeleton edges

# Blatantly False theorem.

refs: Bott & Taubes, Thurston D

D: $E_i(D)$: internal edges
$E_s(D)$: skeleton edges
$V_i(D)$: Internal verts
$V_s(D)$ skeleton verts

$$Z_{-1}(\gamma) = \sum_{D \in [\cdots]} \frac{D}{|\mathrm{Aut}(D)|} \int_{C_D(\mathbb{R}^3,\gamma) \subset (S^1)^{V_s(D)} \times (\mathbb{R}^3)^{V_i(D)}} \prod_{e \in E_i(D)} \Phi_e^* \omega \in \mathcal{D}^{-1}$$

is knot invariant. Furthermore

1. It is a UFTI/Expansion, hence solving the problem to be posed on Monday.
2. It is the perturbative evaluation of the CS QFT, to be defined on Friday.

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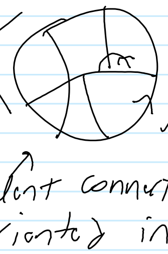
Fixing Thm (-1)

1. The internal edges of D must be oriented
2. The sets $V_i, V_s$ must be ordered

$\mathcal{D}^{-1} = \Big\langle$ v.s. spanned by connected trivalent D's with skeleton $S^1$, oriented edges & ordered $V_i$ & $V_s$ $\Big\rangle \Big/$ For internal edges: $\longrightarrow + \longleftarrow = 0$; re-ordering $V_i / V_s$ acts by the sign of the permutation

done here

Lemma $\mathcal{D}^{-1} \cong \overset{\mathcal{D}^0}{\overset{\shortparallel}{\langle}}$  $\rangle \Big/$ 

Trivalent connected with skeleton $S^1$, unoriented internal edges, unordered $V_i, V_s$, but "oriented internal verts"

Proof ....