Wednesday-2 AKT on 140115: The self-linking number and framings

bring a tube & a laser pointer!

January-14-14 9:27 AM

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1. $l(\gamma_1,\gamma_2)=\sum_{x\in\{\text{xings of } \gamma_1 \,\&\, \gamma_2\}}(-1)^x$

2. $\Phi: T^2\to S^2$ $\Phi(s_1,s_2)=\frac{\gamma_2(s_2)-\gamma_1(s_1)}{\|\quad\|}$ $l(\gamma_1,\gamma_2)=\int_T \Phi^*\omega$, $\int_{S^2}\omega=1$.

on board

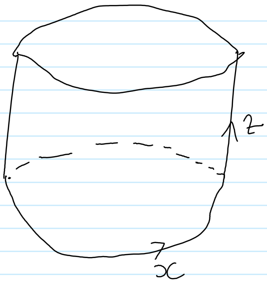
3 The linking number as a degree: Given $\Phi: M^n\to N^n$ between oriented manifolds,

$$\deg\Phi=\frac{\Phi_*[M]}{\Phi_*[N]}=\int_M\Phi^*\omega_N=\sum_{x\in\Phi^{-1}(y)}\deg_x\Phi$$

is a homotopy invariant. $l(\gamma_1,\gamma_2)=\deg\Phi^{\gamma_1,\gamma_2}$

(The emptiest discussion in Mathematics — in two messy ways we will construct a knot invariant in $\mathbb{Z}/\mathbb{Z}$ ....)

$$\eta(\gamma):=\frac{1}{4\pi}\int_{C_2(S^1)}\Phi^*\omega=\frac{1}{4\pi}\int_{\tilde{C}_2(S_1)}\Phi^*\omega$$



$$C_2(S^1)=\{(x,y):\ \begin{matrix}x,y\in S^1\\ x\neq y\end{matrix}\}=S^1_x\times(0,1)_z \qquad y=x+z$$

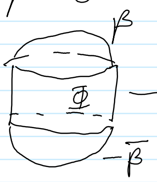
$$\longrightarrow \tilde{C}_2(S^1)=S^1\times[0,1]$$

$$\Phi(x,0):=\lim_{\epsilon\downarrow 0}\frac{\gamma(x+\epsilon)-\gamma(x)}{\|\quad\|}=\frac{\dot\gamma(x)}{\cancel{\|\quad\|}}$$

$$\Phi(x,1):=\ldots \qquad =-\dot\gamma(x)$$

$$0=\int_{\overline{C_2(S^1)}\times I}d\overline{\Phi^*_H}\omega=\int_{\partial(\overline{C_2(S^1)}\times I)}\overline{\Phi^*_H}\omega=\eta_1-\eta_0+2\int_{S^1\times I}\overline{\Phi^*_H}\omega$$

choose a "swaddling map" $\beta: D^2\to S^2$ s.t. $\beta|_{S^1}=\dot\gamma$ and set



$\xrightarrow{\Phi_\beta} S^2$

Post Mortem: Throughout the whole class I should have stuck to framings and avoided swaddlings!

def $sl_1(\gamma,\beta):=\deg\Phi_\beta$
$=\int_{S^2}\Phi_\beta^*\omega\in\mathbb{Z}$

* $sl_1(\bigcirc, \bigcirc)=1$

done line

* in general, $sl_1(\gamma,\beta)$ is an odd integer.

* As a function of γ alone, it is defined up an even integer.

$$\eta(\gamma):=\frac{1}{4\pi}\int\Phi^*\omega$$

$C_2(S^1) \leftarrow$ not compact!

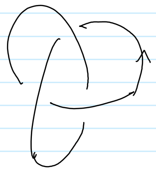
instead consider $sl_2(\gamma,\nu) := l(\gamma,\gamma^{+\nu})$

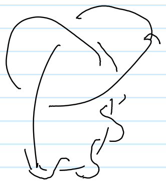
$\gamma^{+\nu}(s) := \gamma(s)+\epsilon\nu(s)$ ... framing, framed knots,

framings $\sim \mathbb{Z}$ as an affine set!

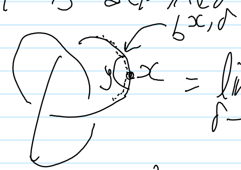
(compare two framings by a map $S^1\to S^1$)

 $\longrightarrow sl_2 = 0$

 in BB framing, $sl_2 = 3$

 0-framing.

Overall, sl_2 is an integer, and as a function of γ alone, it is defined mod $\mathbb{Z}$.

$\eta(\gamma) - sl_2(\gamma,\nu) =$  $= \lim_{\delta\to 0}\int_{S^1_x}\int_{b_y^{x,\delta}}$ (pull back of w) $=$

$= \int_{S^1_x}$ (a local quantity λ computable from γ,ν near x.)

1. if $\nu = \frac{\ddot\gamma}{\|\ \|}$ "the normal of γ" then λ is the Frenet-Serret torsion τ: with $n = \ddot\gamma/\|\ \|$, $\tau = \dot n \cdot (\dot\gamma \times n)$
2. Otherwise, λ is "the drift rel. to the Riemannian connection".

<u>Thm</u> There is a bijection between (homotopy classes of) swaddling maps and odd framings.

If $\beta \leftrightarrow \nu$, then $sl_1(\gamma,\nu) = sl_2(\gamma,\beta)$