Monday-12 AKT on Friday 140404: Back to Expansions

April-04-14 7:39 AM

Today's Monday, there will be a Friday class on Monday.
HW9 due, no HW 10.
For-grade students: discussion after class.
Summer meetings starting next-next-week.

K: a ring, $I \subset K$ an ideal [Typically $K = \mathbb{Q}G$, I: Aug.]

$$\widehat{gr}\, K := K/I \oplus I/I^2 \oplus I^2/I^3 \oplus \dots = \widehat{\bigoplus_{n \geq 0}} I^n/I^{n+1}$$

Expansion: $Z: K \to \widehat{gr}\, K$ s.t. $gr\, Z = Id$, or

$$Z(\gamma \in I^n) = (0, \dots 0, \underset{n}{[\gamma]}, * \dots *)$$

"homomorphic" if respects products.

An A-expansion is:

1. a graded A with a surjective $\pi: A \to \widehat{gr}\, K$

2. $Z: K \to A$ s.t. $\pi /\!/ \widehat{gr}\, Z = I_A$

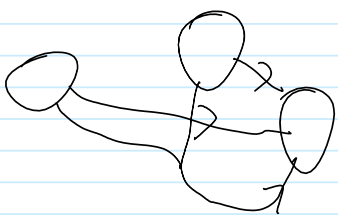
$$Z(\pi(a \in A_n)) = (0, \dots, 0, \underset{n}{a}, * * \dots)$$

$$\begin{tikzcd} & A \arrow[d, "\pi"] \\ K \arrow[ur, "Z"] & gr\, K \end{tikzcd}$$

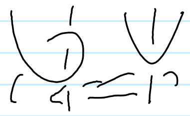
Why good. 1. $A \cong \widehat{gr}\, K$

2. $Z /\!/ \pi$ is an expansion!

Everything extends to general algebraic structures!



Example: FT

Tangles: The category with objects $(\uparrow\downarrow\uparrow\downarrow\uparrow)$
and morphisms  (framed)

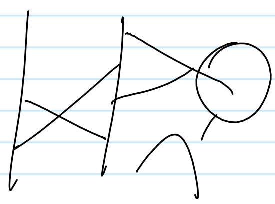
I: The ideal generated by 

I^n: "tangles with n double points"

$(\mathcal{T}/I^{n+1})^*$: invariants of type n.

$(I^n/I^{n+1})^*$: $\mathcal{V}_n/\mathcal{V}_{n+1}$

Guess:

$A = \langle$  $\rangle / 4T$ [describe T!]

Need $Z: \mathcal{T} \to A$ s.t. ...
that's what we have!

.... then some words about homomorphic expansions, associators, etc.