Friday-8 AKT on 140307: Gauge Invariance, Chern-Simons

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Gauge Theory in the simplest case. $\begin{array}{ccc} V & \longrightarrow & V\times M \\ A \text{ v.s.} & & \downarrow \\ & & M \end{array}$

On board

$G = Aut(V) = GL_n \quad \mathfrak{g} = End(V) = M_{n\times n}$

$g \in \tilde{G} = C^\infty(M, G) \circlearrowright \Omega^1(M, \mathfrak{g})$ by

$A \longmapsto A^g = g^{-1}Ag + g^{-1}dg$

For any $D: \Omega^0(M,V) \to \Omega^1(M,V)$ $\quad D \mapsto D^g = g^{-1}Dg$

If $s: M \to V$ $\quad D_A = ds + As \quad D_A \mapsto D_A^g = D_{g^{-1}Ag+g^{-1}dg}$

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physics: "Physics should be gauge invariant"

Math: Could cure $\int e^{i\int A\wedge dA} \int_{\gamma_1} A \int_{\gamma_2} A$ thanks to "invariance".

<u>Def</u> $CS(A) = \int_M tr(A\wedge dA + \frac{2}{3}A\wedge A\wedge A)$ for 3-D M.

<u>Exercise</u> $CS(A^g) = CS(A)$

Infinitesimally, $\delta A = dg + [A, g]$

$\delta CS(A) \sim \int tr\, 2[A,g]\wedge dA + 2(dg)\wedge A\wedge A$
$+ 2[A,g]\wedge A\wedge A$

Do not cover!

Holonomies: Given $\gamma: [0,1] \to M$, $v_0 \in V$, seek

$\tilde{\gamma}: [0,1] \to V$ s.t.

$$\gamma^*(D_A)\tilde{\gamma} = 0 \implies \left(\frac{d}{dt} + A(\dot{\gamma}(t))\right)\tilde{\gamma} = 0$$

If A is Abelian, $\tilde{\gamma}(t) = e^{-\int_0^t ds\, A(\dot{\gamma}(s))} v_0$

Otherwise,
$$\tilde{\gamma}(t) = \underbrace{\left(I - \int_0^t ds_1\, A(\dot{\gamma}(s_1)) + \int_{0\le s_1\le s_2\le t} A(\dot{\gamma}(s_2))A(\dot{\gamma}(s_1)) - \dots\right)}_{hol_\gamma(A)(t)} v_0$$

$$\frac{d}{dt} hol_\gamma(A)(t) = -A(\dot{\gamma}(t))\, hol_\gamma(A)(t)$$

done line

<u>Claim</u> $hol_\gamma(A^g) = g(\gamma(t))^{-1}\, hol_\gamma(A)\, g(\gamma(0))$

<u>Corollary</u> If γ is closed, $tr(hol_\gamma(A))$ is

gauge invariant.

<u>Def</u> $Z_{CS}(\gamma) = \frac{1}{Z} \int e^{\frac{ik}{4\pi} CS(A)} \operatorname{hol}_\gamma(A) \mathcal{D}A$