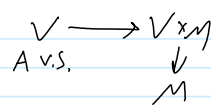


# Friday-8 AKT on 140307: Gauge Invariance, Chern-Simons

March-03-14 8:55 PM

Gauge theory in the simplest case.



$$G = \text{Aut}(V) = GL_n \quad \mathfrak{g} = \text{End}(V) = M_n$$

$$\mathfrak{g} \in \tilde{\mathcal{G}} = C^\infty(M, \mathfrak{g}) \hookrightarrow \mathcal{A}'(M, \mathfrak{g}) \text{ by}$$

$$A \mapsto A^g = g^{-1} A g + g^{-1} dg$$

$$\text{For any } D: \mathcal{U}^\circ(M, V) \rightarrow \mathcal{A}'(M, \mathfrak{g}) \quad D \mapsto D^g = g^{-1} D g$$

$$\text{IF } S: M \rightarrow V \quad D_A = dS + AS \quad D_A \mapsto D_A^g = D_{g^{-1} A g + g^{-1} dg}$$

on board

Physics: "Physics should be gauge invariant"

Math: Could care  $\int_{\gamma_1}^{\gamma_2} e^{i \int A \wedge dA}$  Thanks to "invariance".

Def  $CS(A) = \int_M \text{tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A)$  For 3-D M.

Exercise  $CS(A^g) = CS(A)$  Infinitesimally,  $\delta A = dg + [A, g]$   
 $\delta CS(A) \sim \int 2[A, g] \wedge dA + 2(dg) \wedge A \wedge A + 2[A, g] \wedge A \wedge A$   
 Do not cover!

Holonomies: Given  $\gamma: [0, 1] \rightarrow M$ ,  $v_0 \in V$ , seek

$$\tilde{\gamma}: [0, 1] \rightarrow V \text{ s.t.}$$

$$\gamma^*(D_A) \tilde{\gamma} = 0 \implies \left( \frac{d}{dt} + A(\dot{\gamma}(t)) \right) \tilde{\gamma} = 0$$

IF  $A$  is Abelian,  $\tilde{\gamma}(t) = e^{-\int_0^t A(\dot{\gamma}(s))} v_0$

Otherwise,  $\tilde{\gamma}(t) = \left( I - \int_0^t ds_1 A(\dot{\gamma}(s_1)) + \int_{0 \leq s_1, s_2 \leq t} A(\dot{\gamma}(s_2)) A(\dot{\gamma}(s_1)) - \dots \right) v_0$

$$\underbrace{\hspace{15em}}_{\text{hol}_\gamma(A)(t)}$$

$$\frac{d}{dt} \text{hol}_\gamma(A)(t) = -A(\dot{\gamma}(t)) \text{hol}_\gamma(A)(t)$$

done line

claim  $\text{hol}_\gamma(A^g) = g(\gamma(t))^{-1} \text{hol}_\gamma(A) g(\gamma(0))$

Corollary IF  $\gamma$  is closed,  $\text{tr}(\text{hol}_\gamma(A))$  is

gauge invariant.

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$$\text{Det } Z_{CS}(\chi) = \frac{1}{Z} \int e^{\frac{i\kappa}{4\pi} CS(A)} \text{hol}_g(A) \mathcal{D}A$$