Friday-2 AKT on 140117: Euler-Lagrange, Gaussian Integration

January-16-14 4:40 AM

Last Wed class: In the discussion of degrees the two manifolds involved must be "closed", meaning compact and having no boundary. } forgotten

on board!

Goal for next 2-3 classes:

$$\int_{A\in\Omega^1(\mathbb{R}^3)} \mathcal{D}A\, e^{\frac{i}{4\pi}\int_{\mathbb{R}^3} A\wedge dA} \cdot \int_{\gamma_1} A \cdot \int_{\gamma_2} A = C\langle \gamma_1 | d^{-1} | \gamma_2\rangle = C\cdot l(\gamma_1,\gamma_2)$$

But before, finish "particle on a quantum pendulum" (before actually solving E-L, discuss the relationship with the brachistochrone, power lines, handout browsing and general geodesics, and the rest of physics.

Follow http://drorbn.net/AcademicPensieve/Classes/14-1350-AKT/FridayIntro-Pass2.html

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1. $\int_{-\infty}^{\infty} e^{-x^2/2}\,dx = \sqrt{2\pi}$

2. $\int_{\mathbb{R}^n} e^{-x^2/2} = (2\pi)^{n/2}$

3. Volumes of spheres:

$$(2\pi)^{\frac{n+1}{2}} = \int_{\mathbb{R}^{n+1}} e^{-x^2/2} = \int_0^\infty V_n\, r^n\, e^{-r^2/2}\,dr \Rightarrow V_n = (2\pi)^{\frac{n+1}{2}}/I_n$$

$$I_0 = \tfrac{1}{2}\sqrt{2\pi}$$

$$I_1 = -e^{-r^2/2}\Big|_0^\infty = 1$$

$$I_n := \int_0^\infty r^n e^{-r^2/2}\,dr$$

$$= \int_0^\infty \underbrace{r^{n-1}}_{f}\, \underbrace{r e^{-r^2/2}}_{g'}\,dr = fg\Big|_0^\infty + \int_0^\infty (n-1) r^{n-2} e^{-r^2/2}\,dr$$

$$= (n-1) I_{n-2}$$

$$I_2 = I_0 = \tfrac{1}{2}\sqrt{2\pi} \qquad V_2 = \frac{(2\pi)^{3/2}}{\frac{1}{2}(2\pi)^{1/2}} = 4\pi$$

$$V_1 = 2\pi$$

$$V_0 = (2\pi)^{1/2} / \tfrac{1}{2}\sqrt{2\pi} = 2$$

done line

4. $$\int_{\mathbb{R}} e^{-\frac{\lambda}{2}x^2}\,dx = \sqrt{\frac{2\pi}{\lambda}}$$

5. $$\int_{\mathbb{R}^n} e^{-\frac{1}{2}\lambda_{ij}x^i x^j}\,dx = \int_{\mathbb{R}^n} e^{-\frac{1}{2}\langle x, \Lambda x\rangle}\,dx \qquad \Lambda = (\lambda_{ij}) \text{ positive definite}$$

$$= \frac{(2\pi)^{n/2}}{\det(\Lambda)^{1/2}} = C_\Lambda$$

6. $$\int_{\mathbb{R}^n} P(x)\, e^{-\frac{1}{2}\langle x, \Lambda x\rangle}\,dx = P\left(\frac{\partial}{\partial y_1} \cdots \frac{\partial}{\partial y_n}\right) \int_{\mathbb{R}^n} e^{-\frac{1}{2}\langle x, \Lambda x\rangle + y\cdot x}\,dx \Big|_{y=0}$$

$$= P\left(\frac{\partial}{\partial y}\right) \int_{\mathbb{R}^n} e^{-\frac{1}{2}\langle x-\Lambda^{-1}y, \Lambda(x-\Lambda^{-1}y)\rangle + \frac{1}{2}\langle y, \Lambda^{-1}y\rangle}\,dx \Big|_{y=0}$$

$$= \frac{(2\pi)^{n/2}}{\det(\Lambda)^{1/2}} P\left(\frac{\partial}{\partial y}\right) e^{\frac{1}{2}y\cdot\Lambda^{-1}y}\Big|_{y=0}$$

<u>Example</u> $$\int x^i x^j e^{-\frac{1}{2}\lambda_{ij}x^i x^j}\,dx = \frac{(2\pi)^{n/2}}{(\det\Lambda)^{1/2}} \cdot \lambda^{ij}$$