Algebraic Knot Theory - Splash Page

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Three courses on just one Theorem.

Theorem. $\exists$ expansion

$Z : \mathcal{K}(\uparrow) \longrightarrow \mathcal{A}(\uparrow)$

knots

Something natural, related to knots & to Lie algebras

Prerequisites.

1. Some mathematical maturity
2. Differential forms and Stokes' Theorem at the level of our first-semester core topology class.

Monday Course. Why is this natural, desired, expected, and hard from the perspective of knot theory.

Wednesday course. A proof of the theorem using differential geometry "configuration space integrals"

Friday Course. Where did this pf come from? A very gentle intro to QFT and Feynman diagrams

What's "an expansion"?

A "ring" K, an ideal $I \subset K$,

$$A := I^0/I^1 \oplus I^1/I^2 \oplus I^2/I^3 \oplus \dots$$

An "expansion" is $Z : K \to A$ s.t. if $\gamma \in I^n$ then

$$Z(\gamma) = (0, \dots, 0, \underbrace{\bar{\gamma}}_{\text{in } I^n/I^{n+1}}, *, *, \dots)$$

Example.

K = smooth functions on $\mathbb{R}^n$.

$I = \{f \in K : f(0) = 0\}$

$I^n = \{f : f \text{ vanishes like } |x|^n\}$

$I^n/I^{n+1} \cong$ homogeneous polynomials of degree n

Z is a "Taylor expansion"!

So "Taylor expansions" are vastly general.

So knots can be Taylor-expanded!

Duh? Take the course and see.

... This is but the tip of a huge iceberg...