

Optimistic Rough Tentative Plan. Sixth introduction: meta and beta. Then the full Vietnam story. (was the plan for day 7)

Pasted from <<http://www.math.toronto.edu/~drorbn/Talks/Aarhus-1305/>>

1. The meta/beta story:

Meta-Groups, Meta-Bicrossed-Products, and the Alexander Polynomial, 1
 Dear Bar-Natan in Aarhus, June 2013.
 Abstract: I will define "meta-groups" and explain how one specifies a meta-group, which in itself is a "meta-bicrossed-product", given m_1^* and m_2^* . Quick to compute, but computation depends from topology. Extends to tangles, but at an exponential cost.
 • Hard to categorify.
 Idea. Given a group G and two "YB" pairs $R^1 = (g_1^*, g_1^*) \in G^2$, map them to X and "multiply along", so that $Z \rightarrow Z$.
 This fails! R2 implies that $g_1^* g_2^* = \epsilon = g_2^* g_1^*$ and then R3 implies that g_1^* and g_2^* commute, so the result is a simple counting invariant.
A Group Computer. Given G , can store group elements and perform operations on them:

$$\begin{matrix} x: g_1 & & z: g_2 \\ y: g_2 & \xrightarrow{m_1^*} & \\ & & \\ & & \\ & & \end{matrix}$$

 Also has S_2 for inversion, e_x for unit insertion, d_x for register deletion, Δ_x for element cloning, ρ_x^c for renaming, and $(D_1, D_2) \rightarrow D_1 \cup D_2$ for merging, and many obvious composition actions relating those:
 $P = \{z: g_1; y: g_2\} \rightarrow P = \{d_x P\} \cup \{d_y P\}$
A Meta-Group. Is a similar "computer", only its internal structure is unknown to us. Namely it is a collection of sets (G_i) indexed by all finite sets γ , and a collection of operations m_i^* , S_i , e_x , d_x , Δ_x , ρ_x^c (sometimes), ρ_x^c , and \cup , satisfying the exact same linear properties.
Example 0. The non-meta example, $G_0 := G^2$.
Example 1. $G_0 := M_{n \times n}(\mathbb{Z})$, with simultaneous row and column operations, and "block diagonal" merges. Here if $P = \begin{pmatrix} x & a \\ y & c \end{pmatrix}$ then $d_x P = \begin{pmatrix} x & a \\ y & 0 \end{pmatrix}$ and $d_y P = \begin{pmatrix} x & a \\ y & d \end{pmatrix}$ so $\{d_x P\} \cup \{d_y P\} = \begin{pmatrix} x & a & 0 \\ y & 0 & d \end{pmatrix} \neq P$. So this G is truly meta!
Chain. From a meta-group G and YB elements $R^1 \in G_2$ we can construct a knot/tangle invariant.
Bicrossed Products. If $G = HT$ is a group presented as a product of two of its subgroups, with $H \cap T = \{e\}$, then also $G = TH$ and G is determined by H, T , and the "swap" map $sw^{HT}: (t, h) \mapsto (h, t)$ defined by $th = ht'$. The map sw^{HT} satisfies (1) and (2) below; conversely, if $sw: T \times H \rightarrow H \times T$ satisfies (1) and (2) (+ lesser conditions), then (3) defines a group structure on $H \times T$, the "bicrossed product".
Further meta-bicrossed-products. Π (and variants), \bar{A} (and quotients), \bar{A} (and quotients), $M_0, M, X^{00}, X^{0b}, \dots$
Meta-Lie-algebras. \bar{A} (and quotients), S, \dots
Meta-Lie-bialgebras. \bar{A} (and quotients), \dots
 I don't understand the relationship between gr and H , as it appears, for example, in braid theory.
Meta-business!
mean business!
Some testing
divide and conquer!
Why Happen? Applications to w -knots.
 • Everything that I know about the Alexander polynomial can be expressed cleanly in this language (even if without proof), except HF, but including genus, ribbonness, cabling, w -knots, knotted graphs, etc., and there's potential for vast generalizations.
 • The least wasteful "Alexander for tangles"
 • In aware of.
 • Every step along the computation is the invention of something.
 • Fits on one sheet, including implementation and propaganda.
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Meta-Lie-bialgebras. \bar{A} (and quotients), \dots
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Partial To Do List. 1. Where does it more simply come from?
 2. Remove all the denominators.
 3. How do determinants arise in this context?
 4. Understand links ("meta-conjugacy classes").
 5. Find the "reality condition".
 6. Do some "Algebraic Knot Theory".
 7. Categorify.
 8. Do the same in other natural quotients of the v/w -story.
 "God created the knots, all else in topology is the work of man."
 - David Hume on math
 www.klein.org

$2 \text{ wA}(\Gamma_n) =$ $/ \text{STU}_{1,23}$ AS IHX is a co-commutative Lie algebra.

primitives: diagrams that are connected after removing the skeleton, = wheels and trees

$0 \rightarrow \{\text{wheels}\} \rightarrow \text{wA}(\Gamma_n) \rightarrow \text{trees} \rightarrow 0$

$0 \rightarrow \begin{matrix} tr_n \\ \cong \\ CW_n \end{matrix} \rightarrow \text{wA}(\Gamma_n) \rightarrow \bigoplus_n \text{Lie}(n) \rightarrow 0$

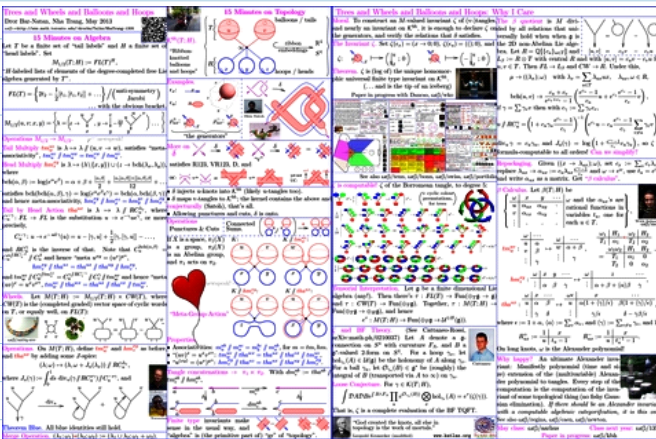
$$\mathbb{C}W_n \xrightarrow{\text{WRT}} \text{LiE}(n) \xrightarrow{\text{WRT}} \text{LiE}(n) \xrightarrow{\text{WRT}} \dots$$

Also in bi-algebras, $\square Z = Z \otimes Z$ is called "group like", & then $Z = \mathbb{C}^3$, } primitive Problem: Compute $\int(K)$.

3. From tangles to balloons & hoops
 "separating the body from the soul"
 "body from shadow".

"knots = balloons & hoops"

4 Z for KBH's ; } is valued in $\text{Lie}(\mathbb{T})^H \times \text{CW}(\mathbb{T})$



$$\begin{array}{ccc}
 FL(T) & \xrightleftharpoons[C_u^{-\gamma}]{RC_u^{\gamma}} & FL(T) \\
 \searrow \phi & & \searrow \bar{\phi} \\
 & & u \rightarrow \bar{u} \\
 & & \left(\begin{array}{c} \bar{u} = e^{-\text{ad } \gamma} u \\ \text{and / or} \\ u = e^{\text{ad } \gamma} \bar{u} \end{array} \right) \\
 FL(T \cup \{\bar{u}\}) & &
 \end{array}$$

$$J_u(\gamma) := \int_0^1 ds \text{div}_u(\gamma // RC_u^{s\gamma}) // C_u^{-s\gamma}.$$