

K = PD[X[1, 5, 2, 4], X[5, 3, 6, 2], X[3, 1, 4, 6]]

PD[X[1, 5, 2, 4], X[5, 3, 6, 2], X[3, 1, 4, 6]]

t1 = K /. X[i_, j_, k_, l_] => AP[i, l] P[j, k] + BP[i, j] P[k, l]

PD[BP[1, 5] P[2, 4] + AP[1, 4] P[5, 2],

AP[3, 6] P[5, 2] + BP[5, 3] P[6, 2], AP[1, 4] P[3, 6] + BP[3, 1] P[4, 6]]

t2 = Times@@t1

(AP[1, 4] P[3, 6] + BP[3, 1] P[4, 6])

(BP[1, 5] P[2, 4] + AP[1, 4] P[5, 2]) (AP[3, 6] P[5, 2] + BP[5, 3] P[6, 2])

t3 = Expand[t2]

A²BP[1, 4] P[1, 5] P[2, 4] P[3, 6]² P[5, 2] +

AB²P[1, 5] P[2, 4] P[3, 1] P[3, 6] P[4, 6] P[5, 2] +

A³P[1, 4]² P[3, 6]² P[5, 2]² + A²BP[1, 4] P[3, 1] P[3, 6] P[4, 6] P[5, 2]² +

AB²P[1, 4] P[1, 5] P[2, 4] P[3, 6] P[5, 3] P[6, 2] +

B³P[1, 5] P[2, 4] P[3, 1] P[4, 6] P[5, 3] P[6, 2] +

A²BP[1, 4]² P[3, 6] P[5, 2] P[5, 3] P[6, 2] +

AB²P[1, 4] P[3, 1] P[4, 6] P[5, 2] P[5, 3] P[6, 2]

t4 = t3 //. P[a_, b_] P[b_, c_] => P[a, c]

AB²P[1, 4]² + B³P[2, 2] P[3, 3] + AB²P[3, 6]² + A²BP[1, 4]² P[3, 6]² + AB²P[5, 2]² +

A²BP[1, 4]² P[5, 2]² + A²BP[3, 6]² P[5, 2]² + A³P[1, 4]² P[3, 6]² P[5, 2]²

t5 = t4 /. {P[a_, a_] => d, P[a_, b_] ^ 2 => d}

3AB²d + 3A²Bd² + B³d² + A³d³

t6 = Expand[Simplify[t5/d /. {B -> 1/A, d -> (-A^2 - 1/A^2)}]]

$-\frac{1}{A^5} - A^3 + A^7$

g