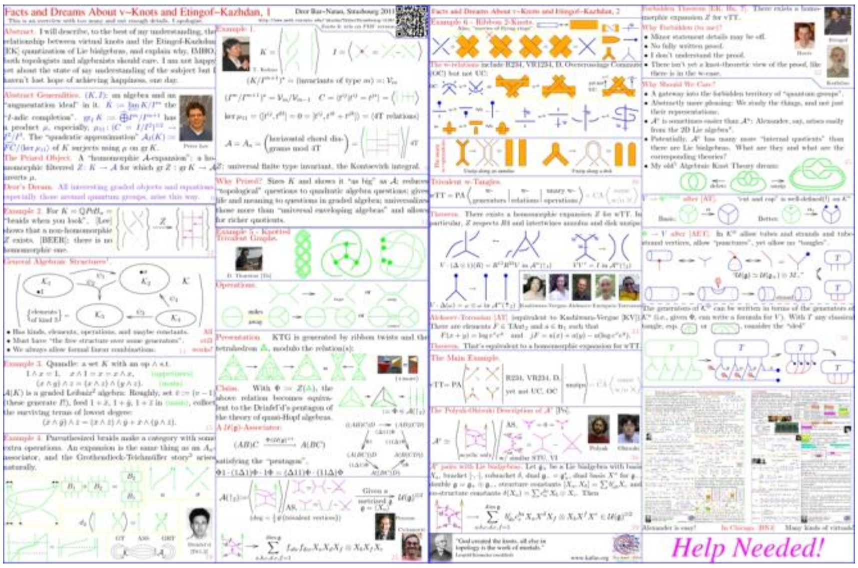
© | Dror Bar-Natan: Classes: 1112: 1100-Algebral: Sep 20:

The Selick Week



Dror's week: http://www.math.toronto.edu/~drorbn/Talks/Strasbourg-1109/

Warnings. For Dror,

1. $x^g = g^{-1}xg$ so that $(x^g)^h = x^{(gh)}$

2. If $\sigma, \tau \in S_n$, then $\sigma\tau = \sigma \circ \tau$

Definitions. Homomorphism, isomorphism, subgroup, cosets, normal subgroup, $C_G(X)$, $Z(G)$, $N_G(X)$.

Proposition. Every normal subgroup is the kernel of a homomorphism & vice versa. (Pf: Define G/N!)

The 1st Isomorphism Thm. If $\phi: G \to H$ is a morphism, then $G/\ker\phi \cong \operatorname{im}(\phi)$

Claim. For $H, K < G$, $HK < G$ iff $HK = KH$.

Claim. If $H \subset N_G(K)$ then $HK = KH$, $K \triangleleft HK$, & $H \cap K \triangleleft H$.

The 2nd isomorphism theorem. If $H < N_G(K)$, then

$$HK/K \cong H/H\cap K$$

$$\begin{array}{ccc} & & HK \\ & \diagup & \wr\!\mid \\ H & & \\ \wr\!\mid & & K \\ H\cap K & \diagup & \end{array}$$

The 3rd Isomorphism Thm. If $K, H \triangleleft G$ & $K < H$, then

$$\frac{G/K}{H/K} \cong G/H$$

The 4th Isomorphism Thm. If $N \triangleleft G$ then $\pi: G \to G/N$ induces a "faithful" bijection between subgroups of G/N and $\{H : N < H < G\}$:

* $A < B \Leftrightarrow \pi(A) < \pi(B)$ (& then, $[B:A] = [\pi(B):\pi(A)]$)
* $A \triangleleft B \Leftrightarrow \pi(A) \triangleleft \pi(B)$
* $\pi(A \cap B) = \pi(A) \cap \pi(B)$.

Permutation Groups. S_n, $|S_n| = n!$, $\text{sign}: S_n \to \{\pm 1\}$ by

$$\text{sign}(\sigma) = (-1)^\sigma = \prod_{i<j} \text{sign}(\sigma j - \sigma i)$$

is a homomorphism, so $A_n := \ker(\text{sign}) \triangleleft S_n$, $|A_n| = \frac{n!}{2}$.

Thm. For $n \neq 4$, A_n is "simple" — it has no normal subgroups except the trivial one and itself.

Thanks, Paul, for teaching for me, and **Parker** for the detailed notes!