October 4, hours 9-10: Simplicity of A_n, Group Actions

October-02-11
7:10 AM

* Agenda: Simplicity of A_n, group actions.
* Makeup class: Thursday at 9AM? (provincial elections day!)

Read Along?

* Go over handouts.

Definition A G-set (left-G-set) $G \times X \to X$

s.t. $(g_1 g_2)x = g_1(g_2 x)$, $ex = x$. Same as $\alpha : G \to S(X)$.

G-sets are a category!

Examples. 1. G itself, under conjugation.

2. Subgroups(G), under conjugation. } not done.

Examples: 1. G/H when H is not-necessarily normal

Sub-example: S_n / S_{n-1} $\quad \sigma S_{n-1} = \sigma' S_{n-1}$ iff

$\sigma(n) = \sigma'(n)$. Let $\tau_i(n) = i$, then

$\sigma \tau_i S_{n-1} = \tau_{\sigma i} S_{n-1}$. So S_n/S_{n-1} is $\{1 \ldots n\}$...

2. If X_1, X_2 are G-sets, then so is $X_1 \sqcup X_2$.

3. $S^2 = SO(3)/SO(2)$

done line

Theorem. 1. Every G-set is a disjoint union of "transitive G-sets"

2. If X is a transitive G set and $x \in X$, then $X \cong G/\mathrm{stab}_X(x)$. (So $|X| \,\big|\, |G|$)

Theorem. If X is a G set and x_i are representatives of the orbits, then

$$|X| = \sum_i \frac{|G|}{|\mathrm{stab}_X(x_i)|}$$

Example. If G is a p-group, the Centre of G is not empty.