October 20, hour 18: Solvable groups, rings

October-18-11
11:30 PM

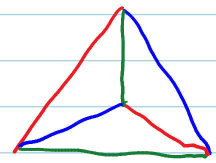
Read Along. Selick 1.11, 2.1

HW2 due. today 11:30–12:30

Term test Tuesday. my OH 10:30 – 12:30 Monday
sthyler OH Mon 5-7 Huron 1028

Riddle Along ?

Agenda 12, Solvable, rings.



Claim $(\mathbb{Z}/2 \times \mathbb{Z}/2) \rtimes \mathbb{Z}/3 \cong A_4$

Solvable Groups. Def G is solvable if all quotients in its Jordan-Hölder series are Abelian.

Thm 1. If $N \triangleleft G$, G is solvable iff N & G/N are.

2. If $H < G$ and G is solvable, so is H.

$A \triangleleft B$ $\quad H \cap A \triangleleft H \cap B$? ✓ $\frac{H \cap B}{H \cap A} \to B/A$ by $[b]_{H\cap A} \to [b]_A$ is injective.

Cor. If a group contains A_n, $n \neq 4$, it is not solvable.

Term test line.

Rings.

Definition 2.1.1. *A* ***ring*** *consists of a set R together with binary operations + and · satisfying:*

1. $(R, +)$ *forms an abelian group,*
2. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ $\forall a, b, c \in R$,
3. $\exists 1 \neq 0 \in R$ *such that* $a \cdot 1 = 1 \cdot a = a$ $\forall a \in R$, *and*
4. $a \cdot (b + c) = a \cdot b + a \cdot c$ *and* $(a + b) \cdot c = a \cdot c + b \cdot c$ $\forall a, b, c \in R$.

Also define: Commutative ring.

done line

Examples. $\mathbb{Z}$, $\mathbb{R}[x]$, $M_{n\times n}(\mathbb{R})$

Morphisms, (Examples: 1. $\mathbb{Z} \to \mathbb{Z}/n$
2. $R \to R[x]$ at deg 0
3. $R \to M_{n\times n}(R)$ as diag
4. $ev_u : R[x] \to R$ (if R is commutative)
5. $M_n(R[x]) \cong M_n(R)[x]$)

(if R is commutative)

5. $M_{n\times n}(R[x]) \cong M_{n\times n}(R)[x]$

im, subring, ker, ideal.

Q. Is every ideal a quotient.

Ans. Define R/I.

Good luck w/ term test!