

September 16, hour 3 - Non Commutative Gaussian Elimination, Homomorphisms, Kernels and Images

September-15-10  
6:53 PM

1. Finish tracing the NCGE handout; along do the S\_4 example. *done line*
2. Go over the "about" handout.
3. Group homomorphisms, the "category" of groups, images and kernels. Example: S\_3 is an image of S\_4, but not a kernel.
4. Normal subgroups, kernels are normal.
5. Question: Is there a normal subgroup of S\_4 which is isomorphic to S\_3?

Example  $\sigma_1 = (123)$   $\sigma_2 = (12)(34)$ , in  $S_4$   
 $\begin{matrix} 11 \\ 2314 \end{matrix}$   $\begin{matrix} 11 \\ 2143 \end{matrix}$

11	I		
12	1	22	I
13	2	23	3
$\sigma_1 = 2314$		$\sigma_{12}^{-1} \sigma_2 = 1342$	I
14	5	4	44
$\sigma_{23} \sigma_{13} = 4132$		$\sigma_{13}^{-1} \sigma_{23} \sigma_{12} = 1423$	I

Feed  $\sigma_1 = 2314 \dots$  Feed @  $\sigma_{12}$

Feed  $\sigma_{12}^2 = 3124 \dots$  Feed @  $\sigma_{13}$

Feed  $\sigma_2 = 2143 \dots$  Feed  $\sigma_{12}^{-1} \sigma_2 = 1342 \dots$  Feed @  $\sigma_{23}$

Feed  $\sigma_{12} \sigma_{23} = 2143 \dots$  Feed  $\sigma_{12}^{-1} \sigma_{12} \sigma_{23} = \sigma_{23} \dots$

No point feeding  $\sigma_{ij} \sigma_{kl}$  if  $ik \neq j$

Feed  $\sigma_{23} \sigma_{12} = 3412 \dots$  Feed  $\sigma_{13}^{-1} \sigma_{23} \sigma_{12} = 1423 \dots$  to  $\sigma_{24}$

Feed  $\sigma_{23} \sigma_{13} = 4132 \dots$  to  $\sigma_{14}$

Feed  $\sigma_{24} \sigma_{12} = 4213 \dots$  Feed  $\sigma_{14}^{-1} \sigma_{24} \sigma_{12} = 1423 \dots$  drop.

$\Rightarrow |G| = 4 \cdot 3 \cdot 1 \cdot 1 = 12$ . Is  $4123 \in G$ ?

Write  $2431$  in terms of  $\sigma_{12}$ .