Boxing Day Handout, 1

December-07-10
6:38 AM

The Best HW Ever. open all the boxes in the Theorem about f.g. modules over a PID, and find that new-style diagonalization is the same as old style.
I Failed. But here are a few hints....

The Final: Tuesday Dec 14, 10AM,
$O(1/3)$ class-proven Theorems,
$O(1/3)$ repeat of HW,
$O(1/3)$ Fresh exercises
} all very approximate
Choose $O(5)$ of $O(6)$, similar to the term exam.

Dror's recommended study. Make sure that you <u>absolutely</u> understand everything that was done in class, worry less about exercising.

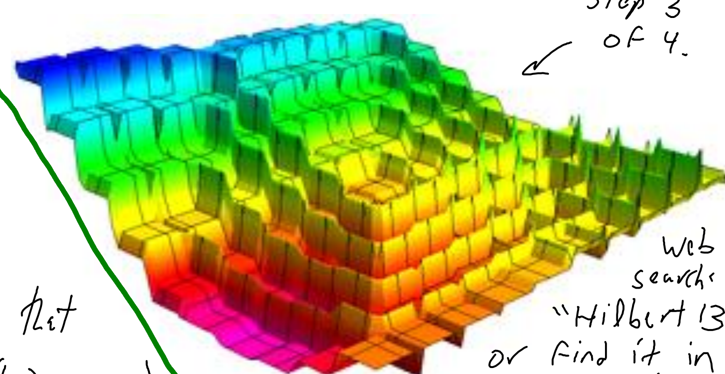
HW5 due on OH, Thursday.

Old style. Given $A \in M_{n\times n}(F)$,

1. Compute $\chi_A(t) = \det(tI - A)$.
   (in practice, must use row & column operations over $R = F[t]$ ....)
2. Let $\lambda_1, \dots, \lambda_n$ be the roots
3. Find v_i s.t. $Av_i = \lambda_i v_i$
4. $C^{-1} = \left(v_1 | \dots | v_n\right)$ $\quad CAC^{-1} = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix} = D$

Who needs analysis? Every cont. function on $\mathbb{R}^n$ is a finite composition of $+$'s & cont. functions of one variable. Hear more in my last MAT 327 Topology class, Wed 2-3PM, Sydney Smith 1070.

step 3 of 4.



web search: "Hilbert 13", or find it in drorbn/Talks.

New style. $V = F^n$ is an R-mod: $(R = F[t])$

$$\begin{tikzcd} R^n \arrow[r, "M"] \arrow[d, "Q_i"'] & R^n \arrow[r, "\pi"] \arrow[d, "P_i"'] & V \arrow[r] \arrow[d, phantom, "\cong"] & 0 \\ R^n \arrow[r, "M_i"] & R^n \arrow[r, "\pi_{1,2}"] & V_{1,2} \arrow[r] & 0 \end{tikzcd}$$

Find invertible $P, Q \in M_{n\times n}(R)$ such that

$$M_1 = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & \prod(t-\lambda_i) \end{pmatrix} = \begin{pmatrix} 1 & & \\ & \ddots & \\ & & \chi_A(t) \end{pmatrix} \quad \& \text{ then } \quad M_2 = \begin{pmatrix} t-\lambda_1 & & \\ & \ddots & \\ & & t-\lambda_n \end{pmatrix} = tI - D$$

Note. $\pi(t^k e_i) = A^k e_i$
Note. $M = tI - A$
(easy but requires a proof)

A Key. Express C, (P_1, Q_1) & (P_2, Q_2) in terms of each other.