- Make sure to cover the material needed for HW7.

on board

* Personal statement.

More than anything else, I see myself as a tour guide, and I actually love the city I am explaining.

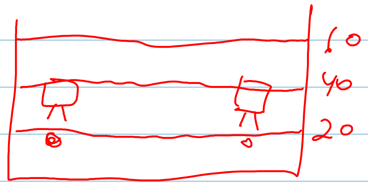
* I tend to spend too much in every street corner, sometimes telling stories that are too intricate and detailed.

* As the end comes near, I'm always sorry for those quarters we didn't explore.

* At the end, you only really understand a city by exploring it yourself.

on board

Just For Fun



how get fastest?

Goals:

* compute ranks * invert. * solve.

on board

Last time: If $A \in M_{m\times n}$ is any matrix, and $P \in M_{m\times m}$ and $Q \in M_{n\times n}$ are invertible matrices, then

$\operatorname{rank} A = \operatorname{rank} PAQ$.

<u>Q1</u> Can we choose P, Q wisely, so as to make PAQ "simpler" than A?

<u>Q2</u> What's simpler?

<u>Ans 2</u>

$$\operatorname{rank}\begin{pmatrix} \overbrace{1 \quad 1 \quad \ddots \quad 1}^{k} & 0 \\ 0 & 0 \end{pmatrix} = \operatorname{rank}\left(\begin{array}{c|c} I_k & 0 \\ \hline 0 & 0 \end{array}\right) = k$$

<u>Ans 1</u> Examples of "good" P/Q: "elementary matrices"

1. Interchanging rows/columns. $E^1_{i,j}$
2. Multiplying r/c by a scalar. $E^2_{i,c}$
3. Adding a multiple of one r/c to another. $E^3_{i,j,c}$

"row/column reduction"

Thm Every matrix A can be r/c-reduced to a block matrix of the form $\begin{pmatrix} I_k & 0 \\ 0 & 0 \end{pmatrix}$.

Example: $A = \begin{pmatrix} 0 & 2 & 4 & 2 & 2 \\ 4 & 4 & 4 & 8 & 0 \\ 8 & 2 & 0 & 10 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{pmatrix}$

Do	Get	Do	Get
1. Bring a 1 to the upper left corner by swapping the first two rows and multiplying the first row (after the swap) by $1/4$.	$\begin{pmatrix} 1 & 1 & 1 & 2 & 0 \\ 0 & 2 & 4 & 2 & 2 \\ 8 & 2 & 0 & 10 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{pmatrix}$	2. Add (-8) times the first row to the third row, in order to cancel the 8 in position 3-1.	$\begin{pmatrix} 1 & 1 & 1 & 2 & 0 \\ 0 & 2 & 4 & 2 & 2 \\ 0 & -6 & -8 & -6 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{pmatrix}$
3. Likewise add (-6) times the first row to the fourth row, in order to cancel the 6 in position 4-1.	$\begin{pmatrix} 1 & 1 & 1 & 2 & 0 \\ 0 & 2 & 4 & 2 & 2 \\ 0 & -6 & -8 & -6 & 2 \\ 0 & -3 & -4 & -3 & 1 \end{pmatrix}$	4. With similar column operations (you need three of those) cancel all the entries in the first row (except, of course, the first, which is used in the canceling).	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 2 & 2 \\ 0 & -6 & -8 & -6 & 2 \\ 0 & -3 & -4 & -3 & 1 \end{pmatrix}$
5. Turn the 2-2 entry to a 1 by multiplying the second row by $1/2$.	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & -6 & -8 & -6 & 2 \\ 0 & -3 & -4 & -3 & 1 \end{pmatrix}$	6. Using two row operations "clean" the second column; that is, cancel all entries in it other than the "pivot" 1 at position 2-2.	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 2 & 0 & 4 \end{pmatrix}$
7. Using three column operations clean the second row except the pivot.	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 2 & 0 & 4 \end{pmatrix}$	8. Clean up the row and the column of the 4 in position 3-3 by first multiplying the third row by $1/4$ and then performing the appropriate row and column transformations. Notice that by pure luck, the 4 at position 4-5 of the matrix gets killed in action.	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

Pasted from <http://katlas.math.toronto.edu/drorbn/index.php?title=06-240/Classnotes_For_Thursday_November_9>

Claim $\operatorname{rank} A = \operatorname{rank}(A^T)$

BTW, the meaning of A^T in the world of l.t. is quite intricate.

claim $\operatorname{rank} A = \dim(\text{col-space}(A)) = \dim(\text{row-space}(A))$

done in full untouched.

Suppose you could row reduce A to I. Find A^{-1}.

$$E_4 E_3 E_2 E_1 A = I \Rightarrow A^{-1} = E_4 E_3 E_2 E_1$$

* The hard way.
* The easy way: r.r. $(A \mid I)$

Example: compute $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1}$.