240Algebra1-091103, Hours 23-24: Linear transformation to matrices, compositions

November-01-09
9:06 PM

* Today's last day to drop this class.

Advance blackboard in blue; today's class in black:

We have an isomorphism (?):

Abstract, general, coord-free — merely numbers, choice-dependent, easy to work with

$$\mathcal{L}(V, W) \longrightarrow M_{m\times n}(F)$$

dim = n $\quad$ m

basis = β $\quad$ γ

$(v_1 \dots v_n)$ $\quad$ $(w_1 \dots w_m)$

$$T \longrightarrow [T]_\beta^\gamma = A$$

$$A = \begin{pmatrix} a_{11} & & & a_{1n} \\ [Tv_1]_\gamma & [Tv_2]_\gamma & \cdots & [Tv_n]_\gamma \\ a_{m1} & & & a_{mn} \end{pmatrix} \Longleftrightarrow T v_j = \sum_{i=1}^{m} a_{kj} w_k$$

Complete the proof that this is a vector space iso.

$$\begin{array}{ccccc} U & \xrightarrow[B]{S} & V & \xrightarrow[A]{T} & W \\ \alpha & & \beta & & \gamma \\ p & & n & & m \end{array}$$

basis: α, β, γ

dim: p, n, m

$T\circ S$: C

If you know A and B, can you find C?

⋮

derive C, then:

Definition $A \in M_{m\times n}$, $B \in M_{n\times p}$ $\quad$ $A\cdot B \in M_{m\times p}$ by

$$(A\cdot B)_{ik} = \sum_{j=1}^{n} A_{ij} B_{jk}$$

Example

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} = \dots$$

Also done: Example: $T_\alpha \circ T_\beta = T_{\alpha+\beta}$ for rotations.

Example: $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} = \dots$

for rotations.

Thm $[T \circ S]_\alpha^\gamma = [T]_\beta^\gamma \cdot [S]_\alpha^\beta$

done in full

stated

Question A matrix $A \in M_{m\times n}$ defines a linear trans $T_A : F^n \to F^m$ by $v \in F^n = M_{n\times 1} \mapsto T_A v = A\cdot v \in M_{m\times 1} = F^m$. What's the matrix corresponding to T_A?

untouched.

The good and the bad about "matrix algebra":

Good	Bad
1. $A+B=B+A$, $(A+B)+C = A+(B+C)$ (basically, all works for addition)	1. Addition is defined only for matrices of same dims.
2. $A(B\cdot C) = (A\cdot B)C$ $\exists I$ s.t. $A\cdot I = A$, $IA = A$	2. mult. is defined only if "mid" dimension matches, & produces an output of yet other dims.
3. If $A\cdot A^{-1} = I$, then $A^{-1}A = I$	3. A^{-1} may not exist even if $A \neq 0$.
4. $(A+B)C = AC + BC$ $A(B+C) = AB + AC$	4. Generally, $A\cdot B \neq B\cdot A$, even when both make sense.

Next goals:
1. Compute rank T/A.
2. Compute A^{-1} (when possible)
3. Solve systems of linear eqn's.