240Algebral-091027, Hours 20-21: Rank Nullity

October-23-09
4:51 PM

TT grades & discussion at end!

VOLUNTEER NOTE - TAKERS
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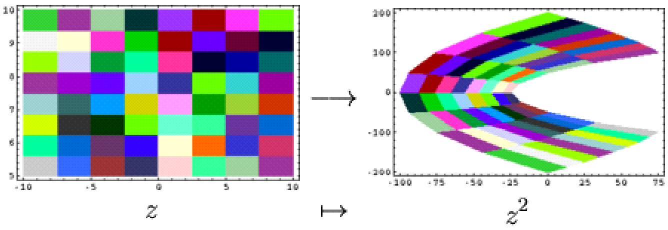
} forgotten

Go over the quilt-plot handout and notebook.

http://katlas.math.toronto.edu/drorbn/index.php?title=06-240/Linear_Algebra_-_Why_We_Care

Linear Algebra is the Small Scale Theory of *Everything* [ec

- **To study the large, start with the small.**
- **In small scales, every space is a vector space.**
  - Indeed if you walk a mile east, a mile north, a mile west and a mile south, you're back were you started, but if you fly a 1,000 miles east, a 1,000 miles north, a 1,000 miles west and a 1,000 miles south, you're not back were you started (where will you be?).
- **In small scales, every function is a linear function.**



(code)

- **The world doesn't come with coordinates.**
  - Hence whenever we can we work without a basis, and when we do study bases, we study all of them.

Fix a l.f. $T: V \to W$

<u>Prop/Def</u> $N(T) = \ker T = \{v : Tv = 0\}$ "null space", "kernel". is a subspace; $\text{nullity}(T) := \dim N(T)$

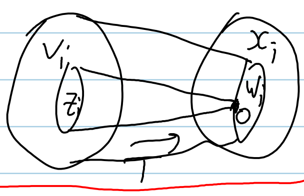
$R(T) = \text{im}\, T = \{Tv : v \in V\}$ "range", "image" is a subspace; $\text{rank}(T) := \dim R(T)$

<u>Thm</u> "the dimension theorem", "the rank-nullity Thm"

Given $T: V \to W$, $\underset{m}{\dim V} = \underset{r}{\text{rank}(T)} + \underset{n}{\text{nullity}(T)}$

<u>Pf</u> $(z_i)_1^n$ basis of $N(T)$, extend to $(z_i) \cup (v_i)$ a basis of V,

done line



<u>claim</u> $w_i := T(v_i)$ are lin. indep. in W Pf....
<u>claim</u> w_i span $R(T)$ Pf...

on board

Corollary of Thm 1 If $\dim V = \dim W$, TFAE

Corollary of thm 1. If $\dim V = \dim W$, TFAE

1. T is 1-1 2. T is onto

3. $\operatorname{rank} T = \dim V$ 4. T is invertible.

Thm 2 $T: V \to W$ & $T': V' \to W'$ are "isomorphic" iff $(\dim V, \dim W, \operatorname{rank} T)$

i.e., $\exists$ a "commutative square of isomorphisms": $= (\dim V', \dim W', \operatorname{rank} T')$

$$\begin{array}{ccc} V & \xrightarrow{T} & W \\ \phi \downarrow & & \downarrow \psi' \\ V' & \xrightarrow{T'} & W' \end{array}$$